Inquisitive Semantics
Student Version

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October 15, 2008

Preliminaries

This paper preceded the Tbilisi paper *Inquisitive Semantics: Two Possibilities for Disjunction*. Also last year, I used it as the main source for the first assignment in the Semantics and Pragmatics course. I left the main contents basically as they were last year, apart from an occasional footnote, and deleting and adding some questions for you to choose to answer.

Although a bit differently organized and presented, the main contents are the same as of the Tbilisi paper. This version is more extended in that it also contains an inquisitive version of predicate logic in an appendix. The last two sections in the Tbilisi paper on Inquisitive Logic and Pragmatics go beyond this version.

1 Mission Statement

In inquisitive semantics, the semantic content of a sentence is not identified with its informative content. Sentences are interpreted in such a way that they can both embody data and issues. And even if a sentence is of a purely informative nature, the semantics will relate it to an issue.

The notion of meaning embodied in inquisitive semantics directly reflects that the primary use of language is communication in dialogue, the exchange of information in a cooperative dynamic process of raising and resolving issues.

The way in which inquisitive semantics enriches the notion of meaning will change our perspective on logic. In the logic that comes with the semantics, the central notion is the notion of licensing.[1] Licensing is concerned with what the utterance of a sentence contributes to a conversation, how it is related to what was said before. Like the standard logical notion of entailment rules the validity of argumentation, the logical notion of licensing rules the coherence of conversation.

The way in which inquisitive semantics enriches the notion of meaning will also change our perspective on pragmatics. The main objective of Gricean pragmatics, is to explain aspects of interpretation which are not directly dictated by semantic content, in terms of general features of rational human behaviour. Since inquisitive semantics changes the notion of semantic content, pragmatics will change with it.

At the heart of Gricean pragmatics is the Cooperation Principle, divided in the Maxims of Quality, Quantity, Relation, and Manner. Conversational implicatures are conclusions one can draw from the utterance of a sentence in a conversation, on the basis of the assumption that the principle and its subsidiary maxims are adhered to.

In inquisitive pragmatics, licensing is the logical twin of the Maxim of Relation. Quality and Quantity will not just involve informativeness, but inquisitiveness as well. This applies also to the derivation of conversational implicatures.

2 The Gricean Picture of Disjunction

In *Indicative Conditionals*, Grice (1989, page 68), Grice gives the following picture of the use of disjunction:[2,3]

> A standard (if not *the* standard) employment of "or" is in the specification of possibilities (one of which is supposed by the speaker to be realized, although he does not know which one), each of which is relevant in the same way to a given topic. 'A or B' is characteristically employed to give a partial answer to some [wh]-question, to which each disjunct, if assertible, would give a fuller, more specific, more satisfactory answer.

[1]I still use here the term 'licensing' rather than 'compliance'. And the notion of 'homogeneity' from the Tbilisi-paper was not invented yet.

[2]I owe the citation to Simons (2000).

[3]The strategy I follow here, is to start from the Gricean picture and 'reason' towards a logical language to deal with it. That brings me to a language where, e.g., the ?-operator should be present, which then later turns out to be definable in terms of disjunction. In the Tbilisi paper it is introduced by definition from the very start.

This is a picture of the (or a) use of disjunction in information exchange. If this is a correct picture, then according to our mission statement, the semantics of disjunction should reflect directly that $(\varphi \vee \psi)$ specifies two possibilities, the possibility that φ and the possibility that ψ. We may add to this – Grice would certainly have agreed – that the semantics of disjunction should certainly do something else as well: exclude the possibility that *neither* φ *nor* ψ. That is the information that $(\varphi \vee \psi)$ provides, its standard truth conditional content.

The Gricean picture also sets a task for a logic that is in accordance with our mission statement: we should be able to account in our logic for Grice's view that φ and ψ should each be relevant to a specific topic, which Grice describes as being a partial answer to some question.

To be able to explicitly do that, we first of all are in need of a logical language in which questions can be expressed. Furthermore, the logic should come with a notion of partial answerhood, and with a notion of contextual licensing with tells us that $(\varphi \vee \psi)$ is contextually licensed iff the context gives rise to a question such that both φ and ψ count as partial answers to that contextual question.

So, the minimal thing we seem to need sofar to model Grice's intuition on disjunction in a logical semantics, is that we specify a propositional language, which contains at least disjunctions and questions. The straightforward way to add questions, is to introduce a sentential operator '?' which turns a sentence φ into a question $?\varphi$.

A standard way to think of the semantic content of a question is that it should specify its possible answers. If we start from an ordinary propositional language, and φ is a sentence in that language, then $?\varphi$ is a polar question, which corresponds to the possibilities that φ and that $\neg\varphi$. This picture of the semantics of questions is not far away from Grice's picture of disjunction. Both disjunctions and questions specify possibilities, but whereas an ordinary disjunction will also exclude possibilities, provides information, a question does not. By the way, in the meantime we also met negation as an element of the language.

What also plays a role in the Gricean picture is *partial* answerhood. If we were to restrict the occurrence of the question operator as the main operator on sentences of an indicative language, we only obtain 'atomic questions' which characteristically have just two (informative) complete answers. If a question specifies a set of possibilities, the notion of partial answerhood that naturally suggests itself, are unions, disjunctions, of complete answers. But if there are only two possibilities, the only such union we get is a non informative tautological partial answer.

An easy way to make room for questions with partial answers is to allow for conjunctions of questions $(?\varphi \wedge ?\psi)$, where a complete answer is the conjunction of a complete answer to $?\varphi$ and a complete answer to $?\psi$. A complete answer to just $?\varphi$ could now count as a partial answer to $(?\varphi \wedge ?\psi)$, and so would the disjunction of a complete answer to $?\varphi$ and a complete answer to $?\psi$.

So, conjunction has entered as an ingredient of the propositional logical language as well, both between questions, and between answers, i.e., between assertions. Natural language uses one and the same word 'and' to conjoin questions and assertions, and there is no reason to assume that it has a different meaning in these two cases. This suggest that we have a single clause in our logical syntax to deal with all cases, and a single interpretation for conjunction in the semantics that gives the right result for all cases.

Actually, since in natural language we also find hybrid conjunctions, like 'John will come to the party, but will Mary come as well?', it makes sense to make the syntactic and semantic rule for conjunction insensitive to the nature of the conjuncts.

Once we have decided to do so for conjunction, even though it may be less obvious for disjunctions, let's decide for the sake of uniformity, that we do the same with disjunction: allow disjunction of sentences freely, give a uniform interpretation rule for that, and see what comes out. If we get unacceptable results, we can always still change this policy.

There is one standard logical operation that sofar remained out of the picture: implication. It makes sense to include it, not only for assertive implication, but also to model conditional questions like 'If Mary comes to the party, will John come as well?', which would naturally translate as a formula of the form $(\varphi \rightarrow ?\psi)$. Following Velisseratou (2000), we take it that our example conditional question has two complete answers, corresponds with two possibilities: the possibility that $(\varphi \rightarrow \psi)$ and the possibility that $(\varphi \rightarrow \neg\psi)$.

Here too, we will go by the assumption that underlying both types of implications is a single interpretation for the implication sign, that is to give correct results in all cases. And although perhaps there are no immediate examples from natural language that suggest the necessity of that, the logical syntax is simplest if we also allow for the antecedent to be of arbitrary nature.

Finally, Grice leaves room for a less standard use of disjunction that does not fit the picture. Let's assume this less standard use of disjunction has nothing to do with specifying possibilities, but just excludes the possibility that neither disjunct holds. Rather than making the logical operation of disjunction ambiguous, let's agree on adding a sentential operator '!' to the language that has to have the effect that $!(\varphi \vee \psi)$ boils down to just its standard truth functional meaning.

So, based on our wish to model what is involved in the Gricean picture of disjunction, and some general deliberations concerning logical simplicity and elegance, we decided that our propositional language is to contain the three sentential operators: $\neg, ?, !$, and the three connectives $\vee, \wedge, \rightarrow$.

Question 1 Mandy Simons (2000) also uses the citation from Grice to start her story (chapter 2 of her dissertation, see Blackboard). So, naturally, there must be a lot in there that is immediately related to inquisitive semantics. A good project to do is search for correspondences and differences between the two stories. For the larger part she deals with pragmatic issues, to which we will only turn after the midsemester break. But a good overview of what she has been doing might be helpful for steering our own investigations.

3 Inquisitive Propositional Syntax

A standard way to look upon an inquisitive syntax is as a syntax for a language in which not only assertions, but also questions can be expressed. Such a language is often called a 'query language'. What you normally expect of such a language is that is has two basic syntactic categories: indicatives and interrogatives.[4]

A distinctive feature of our inquisitive syntax is going to be that it does not make such a syntactic distinction. There is a single category of sentences of the language. The inquisitive semantics for the language will enable us to distinguish assertions and questions among the sentences of the language. Moreover, there will turn out to be a third type of sentences: hybrids, which are neither assertions nor questions, but something in between.

Definition 1 (Inquisitive Propositional Syntax) Let $\wp$ be a finite set of propositional variables. The set of sentences of $L_\wp$ is the smallest set such that:

1. If $p \in \wp$, then $p \in L_\wp$
2. $\bot \in L_\wp$
3. $\top \in L_\wp$

[4] The first order query language defined in Groenendijk [1999] exemplifies such a syntax. The same holds for Velissaratou (2000).

4. If $\varphi \in L_\wp$, then $\neg\varphi \in L_\wp$
5. If $\varphi \in L_\wp$, then $!\varphi \in L_\wp$
6. If $\varphi \in L_\wp$, then $?\varphi \in L_\wp$
7. If $\varphi \in L_\wp$ and $\psi \in L_\wp$, then $(\varphi \to \psi) \in L_\wp$
8. If $\varphi \in L_\wp$ and $\psi \in L_\wp$, then $(\varphi \wedge \psi) \in L_\wp$
9. If $\varphi \in L_\wp$ and $\psi \in L_\wp$, then $(\varphi \vee \psi) \in L_\wp$

The syntax allows us, e.g., to form sentences like $(p \vee q)$, $!(p \vee q)$, and $?(p \vee q)$. The syntax as such makes no syntactic distinction between assertions and questions, but we will be able to distinguish the two in semantic terms. As all sentences preceded by an exclamation mark, $!(p \vee q)$ will count as an assertion; as all sentences preceded by a question mark, $?(p \vee q)$ will count as a question; and $(p \vee q)$, which corresponds with a Gricean disjunction, will count as neither an assertion nor a question, but as a hybrid.

Being preceded by ! is sufficient, but not necessary for a sentence to be an assertion. E.g., atomic sentences are assertions, and so are all negations, sentences of the form $\neg\varphi$. This means that $\neg?\varphi$ is an assertion, but not an informative assertion, but a contradiction, i.e., a sentence equivalent with $\bot$.

Neither is it necessary for a sentence to be preceded by ? to count as a question. The question mark may also occur embedded, as in $(p \to ?q)$, a conditional question, and in $(?p \wedge ?q)$, a conjunction of questions, and in $(?p \vee ?q)$, a disjunction of questions which leaves you the choice of answering either $?p$ or $?q$. The disjunction of conjunctions of questions $((?p \wedge ?q) \vee (?p \wedge ?r) \vee (?q \wedge ?r))$ is a question which leaves you the choice of answering two of the three questions $?p$, $?q$ and $?r$.

The occurrence of a question mark somewhere in the sentence is not sufficient to turn the whole sentence into a question. E.g., $(p \wedge ?q)$ will be a hybrid sentence; $(?p \to q)$ will be an assertion which might be paraphrased as: whether p is the case or not, q is the case. And $(?p \to ?q)$ will be a question, which might be paraphrased as: Does whether q depend on whether p? Given any answer to whether p, what is in that case the answer to whether q?

As compared to standard propositional logic, only clause 5 and 6, introducing the exclamation mark and the question mark, are non-standard. Remarkably enough, the inquisitive semantics we will give for the language will make clear that these two clauses are dispensable: $!\varphi$ will turn out to be equivalent with $\neg\neg\varphi$ and $?\varphi$ with $(\varphi \vee \neg\varphi)$.[5]

Question 2 I don't know whether this is the right point to put this question. But of course, a very general and in the end crucial issue is how the logical syntax and semantics relate to natural language. From the natural language point of view: how sensible is it that we view questions as basically being disjunctions? Has the difference between $(p \vee q)$ and $!(p \vee q)$ in any way a counterpart in natural language? On the more positive side: one can try to find nice natural language examples that *do* exhibit features that the logical language shows.

At certain other points the syntax as defined above could be formulated more sparsely without a loss of expressiveness: as usual, $\top$ will be equivalent with $\neg\bot$, and $\neg\varphi$ with $(\varphi \to \bot)$. Hence, clause 3 and 4 are not essential either. We can do with the five clauses: 1, 2, 7, 8, 9.

A natural question to ask is whether we can do with even less, can we economize on the two-place connectives? In classical propositional logic we can. There $(\varphi \wedge \psi)$ is equivalent with $\neg(\neg\varphi \vee \neg\psi)$, and $(\varphi \vee \psi)$ with $(\neg\varphi \to \psi)$. This means that in classical propositional logic, just having $\bot$ and $\to$ is sufficient to express all meanings (truth functions). In a similar vein, $\neg$ with either $\vee$ or $\wedge$ suffice in classical logic as well.

In inquisitive logic things are different, none of the standard equivalences which make it possible to define one connective in terms of negation and another connective, will hold. At the same time, what does hold is that all meanings that can be expressed with the full language, can also be expressed by only using disjunction and negation. A remarkable situation.

Another interesting thing is that we can consider certain sublanguages of the language as defined above, which do not have the full expressive power of the whole language, but characterize interesting subclasses of the set of all meanings expressible by the full language. For example, $?$, $\neg$ and $\wedge$ give rise to what might be called classical inquisitive propositional logic, where questions correspond to partitions of logical space. Adding $\to$ to that, a richer

[5] I don't take these equivalences to be essential features. I can imagine an interpretation of the language where the operators ! and ? are basic, and would not fully correspond in meaning to double negation and 'tautological disjunction'. Such a different interpretation might then help to account for the fact why certain constructions the present set-up allows for, such as the negation of questions, do not occur in natural language. I don't consider the fact that $\neg?\varphi$ is a contradiction to be a suitable explanation for that.

set of meanings can be expressed, including the meanings of conditional questions.

To be able to really look into such issues, we now turn to the inquisitive semantics.

4 Inquisitive Semantics

We formulate the semantics for an inquisitive language $L_\wp$ by a recursive definition of the notion $\langle i, j \rangle \models \varphi$, where i and j are valuation functions for the atomic sentences in $\wp$, which we call indices.[6] We pronounce $\langle i, j \rangle \models \varphi$ as: $\langle i, j \rangle$ supports φ.

Definition 2 (Indices) Let $L_\wp$ be an inquisitive propositional language.

1. An *index for* $L_\wp$ is a function i such that for all $p \in \wp$: $i(p) \in \{0, 1\}$.

2. The *indices for* $\wp$, $I_\wp$ is the the set of all indices for $\wp$.

Question 3 In G&S handbook article on questions (section 4.4) there is an explicit argument to the effect that the semantics of questions must be intensional. This view has been challenged in a paper by Nelken and Francez (see references, but I'll also put it on Blackboard) by presenting a many-valued extensional (according to them) semantics. It bears some resemblances to what happens in inquisitive semantics. It might be good to have a look at that and spot correspondences and differences. There is also a paper by Nelken and Shan (see references, but I'll also put it on Blackboard) who deal with questions by means of a modal logic. If I remember correctly, they need only two possible worlds to state the semantics. Well, that is very close to what is happening in inquisitive semantics. We also state the semantics relative to two indices, which are much the same sort of thing as possible worlds. Our semantics is also minimally intensional. By the way, there is also a cryptic remark in the Ten Cate and Shan paper (p. 69) that "to test a LoI entailment, it suffices to consider structures with only two possible worlds". So, there is a lot that one could relate to.

[6]Indices are just possible worlds, as in the Tbilisi paper. The reason for not calling them worlds here has to do with the extension to predicate logic that I present at the end. There worlds do no longer suffice, I have to pair them with assignments of values to variables, then 'index' is a more suitable term for such pairs. Using hat term already in the propositional case avoids that I have to restate many definitions. Most definitions of logical notions for the propositional case can remain the same.

Definition 3 (Inquisitive Semantics) Let $\varphi \in L_\wp$, $\langle i, j\rangle \in I_\wp$.

1. $\langle i, j\rangle \models p$ iff $i(p) = 1$ and $j(p) = 1$
2. $\langle i, j\rangle \models \top$
3. $\langle i, j\rangle \not\models \bot$
4. $\langle i, j\rangle \models \neg\varphi$ iff $\langle i, i\rangle \not\models \varphi$ and $\langle j, j\rangle \not\models \varphi$
5. $\langle i, j\rangle \models\ !\varphi$ iff $\langle i, i\rangle \models \varphi$ and $\langle j, j\rangle \models \varphi$
6. $\langle i, j\rangle \models\ ?\varphi$ iff $\langle i, j\rangle \models \varphi$ or $\langle i, i\rangle \not\models \varphi$ and $\langle j, j\rangle \not\models \varphi$
7. $\langle i, j\rangle \models (\varphi \vee \psi)$ iff $\langle i, j\rangle \models \varphi$, or $\langle i, j\rangle \models \psi$
8. $\langle i, j\rangle \models (\varphi \wedge \psi)$ iff $\langle i, j\rangle \models \varphi$ and $\langle i, j\rangle \models \psi$
9. $\langle i, j\rangle \models (\varphi \to \psi)$ iff for all $\iota \in \{i, j\}^2$: if $\iota \models \varphi$, then $\iota \models \psi$

We will not systematically run down the clauses of the definition to explain and illustrate them. We will do so in the course of a longer story in which we introduce some new semantical and logical notions, and note some facts related to them.

4.1 Equivalence and the Dispensability of ? and !

First we introduce the notion of logical equivalence:

Definition 4 (Equivalence)
φ and ψ are *equivalent*, $\varphi \Leftrightarrow \psi$ iff for all $\iota \in I^2$: $\iota \models \varphi$ iff $\iota \models \psi$

And, as announced, we note the following fact:

Fact 1 (Dispensability of ? and !)

1. $?\varphi \Leftrightarrow (\varphi \vee \neg\varphi)$
2. $!\varphi \Leftrightarrow \neg\neg\varphi$

Both the dispensability of ? and of ! follow immediately from the definition of the semantics.

Concerning the dispensability of ?, the definition tells us that $\langle i, j\rangle \models ?\varphi$ iff $\langle i, j\rangle \models \varphi$, or $\langle i, i\rangle \not\models \varphi$ and $\langle j, j\rangle \not\models \varphi$. Concerning the second disjunct, the clause for negation tells us that $\langle i, i\rangle \not\models \varphi$ and $\langle j, j\rangle \not\models \varphi$ iff $\langle i, j\rangle \models \neg\varphi$. So, $\langle i, j\rangle \models ?\varphi$ iff $\langle i, j\rangle \models \varphi$ or $\langle i, j\rangle \models \neg\varphi$. The clause for disjunction tells us that $\langle i, j\rangle \models \varphi$ or $\langle i, j\rangle \models \neg\varphi$ iff $\langle i, j\rangle \models (\varphi \vee \neg\varphi)$. And we are done.

Concerning the dispensability of !, the definition tells us that $\langle i, j\rangle \models \neg\neg\varphi$ iff $\langle i, i\rangle \not\models \neg\varphi$ and $\langle j, j\rangle \not\models \neg\varphi$. Since according to the definition $\langle i, i\rangle \models \neg\varphi$ iff $\langle i, i\rangle \not\models \varphi$, we know that $\langle i, i\rangle \not\models \neg\varphi$ iff $\langle i, i\rangle \models \varphi$, and similarly that $\langle j, j\rangle \not\models \neg\varphi$ iff $\langle j, j\rangle \models \varphi$. Hence, $\langle i, j\rangle \models \neg\neg\varphi$ iff $\langle i, i\rangle \models \varphi$ and $\langle j, j\rangle \models \varphi$. And this means that $!\varphi$ is equivalent with $\neg\neg\varphi$.□

Basically, this tells us that in discussing the semantics, we need not give special attention to $?\varphi$ and $!\varphi$.

4.2 Inquisitiveness, Informativeness and Disjunction

The basic feature of disjunction, is that it introduces inquisitiveness in the language. Inquisitiveness and informativeness are defined as follows:[7]

Definition 5 (Inquisitiveness and Informativeness) Let $\varphi \in L$.

1. φ is *inquisitive* iff for some $i \in I$ and $j \in I$: $\langle i, i\rangle \models \varphi$ and $\langle j, j\rangle \models \varphi$ and $\langle i, j\rangle \not\models \varphi$
2. φ is *informative* iff for some $i \in I$ and $j \in I$: $\langle i, i\rangle \models \varphi$ and $\langle j, j\rangle \not\models \varphi$

We will see later that the meaning of φ gives rise to (at least) two *possibilities* – in the sense in which Grice uses this notion in his picture of disjunction – in case there are indices i and j such that $\langle i, i\rangle \models \varphi$ and $\langle j, j\rangle \models \varphi$ whereas $\langle i, j\rangle \not\models \varphi$, i.e., in case φ is inquisitive. This holds, e.g., in case φ is a disjunction like $(p \vee q)$.

Fact 2 (Inquisitiveness and Informativeness)

1. $(p \vee q)$ is inquisitive and informative.
2. $!(p \vee q)$, i.e., $\neg\neg(p \vee q)$, is informative but not inquisitive.

[7]Note that, unlike in the Tbilisi paper, informativeness is defined in such a way that it implies consistency.

3. $(p \vee \neg p)$, i.e., $?p$, is inquisitive but not informative.

4. $!(p \vee \neg p)$, i.e., $!?p$, is neither inquisitive, nor informative.

5. $\neg(p \vee \neg p)$, i.e., $\neg?p$, is neither inquisitive, nor informative.

6. φ is neither inquisitive nor informative iff $\varphi \Leftrightarrow \top$ or $\varphi \Leftrightarrow \bot$

Consider $(p \vee q)$. Let i and j be indices such that $i(p) = 1$, $i(q) = 0$; and $j(p) = 0$, $j(q) = 1$. Then whereas $\langle i, i \rangle \models (p \vee q)$, because $\langle i, i \rangle \models p$; and $\langle j, j \rangle \models (p \vee q)$, because $\langle j, j \rangle \models p$, we have that $\langle i, j \rangle \not\models (p \vee q)$, because $\langle i, j \rangle \not\models p$ and $\langle i, j \rangle \not\models q$. Hence, $(p \vee q)$ is inquisitive.

$(p \vee q)$ is also informative. Let i be as above, and let k be an index such that $k(p) = 0$, $k(q) = 0$. Then, wheras $\langle i, i \rangle \models (p \vee q)$, we have that $\langle k, k \rangle \not\models (p \vee q)$, since $\langle k, k \rangle \not\models p$ and $\langle k, k \rangle \not\models q$.

Consider $!(p \vee q)$. Let i and k be as above. The clause for ! tells us that $\langle i, i \rangle \models !(p \vee q)$ iff $\langle i, i \rangle \models (p \vee q)$, which we have seen to be the case. Similarly, $\langle k, k \rangle \models !(p \vee q)$ iff $\langle k, k \rangle \models (p \vee q)$, which we have seen not to be the case. So, like $(p \vee q)$, $!(p \vee q)$ is informative.

However, unlike $(p \vee q)$, $!(p \vee q)$ is not inquisitive. Suppose $\langle i, i \rangle \models !(p \vee q)$ and $\langle j, j \rangle \models !(p \vee q)$. As we just saw, this means that $\langle i, i \rangle \models (p \vee q)$ and $\langle j, j \rangle \models (p \vee q)$. By the definition of !, this also means that $\langle i, j \rangle \models !(p \vee q)$.

Consider $(p \vee \neg p)$. For every index i, either $i(p) = 1$ or $i(p) = 0$. Hence, either $\langle i, i \rangle \models p$, or $\langle i, i \rangle \models \neg p$. Then also for every index i: $\langle i, i \rangle \models (p \vee \neg p)$. So $(p \vee \neg p)$ is not informative. There are indices i and j such that $i(p) = 1$ and $j(p) = 0$. For two such indices $\langle i, j \rangle \not\models (p \vee \neg p)$, because $\langle i, j \rangle \not\models p$ and $\langle i, j \rangle \not\models \neg p$, which means that $(p \vee \neg p)$ is inquisitive.

Consider $!(p \vee \neg p)$ and $\neg(p \vee \neg p)$. As we saw above, for every i: $\langle i, i \rangle \models (p \vee \neg p)$. This means that for every i and j: $\langle i, i \rangle \models (p \vee \neg p)$ and $\langle j, j \rangle \models (p \vee \neg p)$. By the definition of ! and $\neg$ that also means that for every i and j: $\langle i, i \rangle \models !(p \vee \neg p)$, and for no i and j: $\langle i, i \rangle \models \neg(p \vee \neg p)$. Hence, $!(p \vee \neg p)$ is equivalent with $\top$, and $\neg(p \vee \neg p)$ is equivalent with $\bot$, and both are neither informative nor inquisitive.□

Apart from telling us that disjunction behaves in a special way, the fact that $(p \vee q)$ and $\neg\neg(p \vee q)$ have different semantic properties, also indicates that the law of double negation does not generally hold in inquisitive semantics; and the fact that $(p \vee \neg p)$ is not a tautology indicates that the law of the excluded middle does not generally hold in inquisitive semantics.

4.3 Questions, Assertions, and Hybrids

The following definition tells us that $(p \vee q)$ is a hybrid, that $!(p \vee q)$ is an informative assertion, and that $(p \vee \neg p)$, and hence $?p$ is an inquisitive question.

Definition 6 (Questions, Assertions and Hybrids) Let $\varphi \in L$.

1. φ is an *(inquisitive) question* iff φ is (inquisitive and) not informative.

2. φ is an *(informative) assertion* iff φ is (informative and) not inquisitive.

3. φ is a *hybrid* iff φ is inquisitive and informative.

The tautological and contradictory cases, such as $!?p$ and $\neg?p$ are questions as well as assertions according to the definition (and not hybrids), but they are neither informative assertions nor inquisitive questions.

Given the way in which questions and assertions are defined, we can note the following handy fact:

Fact 3 (Questions and Assertions) For all $\varphi \in L$:

1. $!\varphi$ is an assertion;

2. $?\varphi$ is a question.

$!\varphi$ can never be inquisitive. By definition $\langle i, j\rangle \models\ !\varphi$ iff $\langle i, i\rangle \models \varphi$ and $\langle j, j\rangle \models \varphi$. Also by definition, $\langle i, i\rangle \models \varphi$ iff $\langle i, i\rangle \models\ !\varphi$, and similarly for $\langle j, j\rangle \models \varphi$. Hence, for all i and j: $\langle i, j\rangle \models\ !\varphi$ iff $\langle i, i\rangle \models\ !\varphi$ and $\langle j, j\rangle \models\ !\varphi$, which implies that $!\varphi$ is not inquisitive, and hence is an assertion.

$?\varphi$ can never be informative: for all i: $\langle i, i\rangle \models\ ?\varphi$. By definition, $\langle i, i\rangle \models\ ?\varphi$ iff $\langle i, i\rangle \models \varphi$ or $\langle i, i\rangle \not\models \varphi$. Given the way in which the semantics is formulated, for every pair $\langle i, j\rangle$ it is always decided whether $\langle i, j\rangle \models \varphi$ or not.$\Box$

Question 4 Prove the following facts:

Fact 4 (Iteration of ? and !) For all $\varphi \in L$:

1. $??\varphi \Leftrightarrow\ ?\varphi$

2. $!!\varphi \Leftrightarrow\ !\varphi$

Fact 5 (Division in Theme and Rheme) For all $\varphi \in L$:

$$\varphi \Leftrightarrow (?\varphi \wedge !\varphi)$$

[Question ends here]

What division says is that for any sentence φ we can factor out a question $?\varphi$ and an assertion $!\varphi$. The assertion that is factored out, wich we call the *rheme* of the sentence, corresponds to the information content of the sentence, the data it provides. The question that is factored out, which we call the *theme* of the sentence, does not just correspond to the issues the sentence raises, but is rather a presupposed *background question* to which the assertive part is an answer.

We are not going to dwell upon this now, but division is obviously related to many hotly debated issues, like topic and focus, information structure, and presupposition.[8]

4.4 Picturing Meanings

We have seen some interesting features of certain sentences, such as that $(p \vee q)$ and $!(p \vee q)$, are not equivalent, that they differ in meaning, but that does probably not yet give us a clear picture of what meanings are assigned to these sentences by the semantics. So, let's first define what the meaning of a sentence is.

Definition 7 (Meanings) Let $\varphi \in L_{\wp}$ and I the set of indices for $\wp$.

The *meaning* of φ, $\langle\varphi\rangle^I = \{\iota \in I^2 \mid \iota \models \varphi\}$.

So, following a standard pattern of defining meanings, we identify the meaning of a sentence with the set of pairs of indices which support it.

Meanings being sets of pairs of indices, we can look upon a meaning as a relation on the set of indices. The way to look upon the relation is to view

[8]Division tells us that $\varphi \to \psi$ is equivalent with $?(\varphi \to \psi) \wedge !(\varphi \to \psi)$. I show in the Tbilisi paper that it also holds for $\varphi \to \psi$ and $(\varphi \to ?\psi) \wedge (\varphi \to !\psi)$. So, there is no unique way to divide a sentence in a theme and a rheme. (The two conjunctions are not necessarily equivalent.) Also, sentences which have different focus elements, should be assigned different themes, should presuppose different issues. Compare the illustrations in *The Logic of Interrogation*.

it as a relation of *indifference*. If two indices are related in the meaning of a sentence, it means that the sentence does not express an interest in the difference between the two. Conversely, if two indices are not related, then the sentence embodies an *issue* concerning the difference between them.

We will see later that such relations which count as meanings have certain properties, and that these properties differ for the three types of sentences we distinguished: assertions, questions and hybrids. But before we turn to that, let us inspect the meanings of $(p \vee q)$ and $!(p \vee q)$.

We saw above that both kinds of disjunction are informative, which implies that for some index $i \in I$: $\langle i, i\rangle \notin \langle(p \vee q)\rangle^I$ and $\langle i, i\rangle \notin \langle !(p \vee q)\rangle^I$. In general, $\langle\varphi\rangle^I$ will be a relation on a *subset* of I, a non-empty proper subset in case φ is informative. Which subset it is, which indices are included and excluded, determines the *informative content* of φ, the *data* it provides. For both $(p \vee q)$ and $!(p \vee q)$ only indices i where $i(p) = i(q) = 0$ are excluded.

Where $(p \vee q)$ and $!(p \vee q)$ differ is which of the included indices are related to each other. In showing that $!(p \vee q)$ is not inquisitive, we saw that $\langle i, j\rangle \models\, !(p \vee q)$ iff $\langle i, i\rangle \models\, !(p \vee q)$ and $\langle j, j\rangle \models\, !(p \vee q)$. This means that in $\langle !(p \vee q)\rangle^I$ *all* indices which are not excluded are related to each other. This holds in general for non-inquisitive sentences. If φ is not inquisitive, if φ is an assertion, then $\langle\varphi\rangle^I$ is a total relation on a subset of I. And if φ is an informative assertion, then $\langle\varphi\rangle^I$ is a total relation on a non-empty proper subset of I.

In contrast, in showing that $(p \vee q)$ is inquisitive, we found that we may have that $\langle i, i\rangle \models (p \vee q)$ and $\langle j, j\rangle \models (p \vee q)$, whereas at the same time $\langle i, j\rangle \not\models (p \vee q)$. This is the case precisely when $i(p) \neq j(p)$ and $i(q) \neq j(q)$. I.e, as long as $i(p) = j(p) = 1$ or $i(q) = j(q) = 1$, i and j are related to each other, i.e., then $\langle i, j\rangle \in \langle(p \vee q)\rangle^I$.

So, we can distinguish two sets of indices P and Q in the meaning of $(p \vee q)$, such that in P all indices are collected where $i(p) = 1$, and in Q all indices are collected where $i(q) = 1$. P and Q overlap: in case $i(p) = i(q) = 1$, then $i \in P$ and $i \in Q$. Being sets of indices, we can look upon P and Q as propositions, the proposition expressing that p and the proposition expressing that q, respectively.

Hence, we have modeled Gricean disjunction: we can look upon P and Q as the two possibilities specified by the disjunction $(p \vee q)$.

It may be worthwhile to note that we have in general that $\langle(\varphi \vee \psi)\rangle^I = \langle\varphi\rangle^I \cup \langle\psi\rangle^I$. We arrive at the meaning of a disjunction, by taking the union of the meanings of its disjuncts. Similarly: $\langle(\varphi \wedge \psi)\rangle^I = \langle\varphi\rangle^I \cap \langle\psi\rangle^I$. The meaning of a conjunction is obtained by taking the intersection of the meanings of the conjuncts. In this, the interpretation of disjunction and conjunction follows a standard pattern. In the following definition, we explicitly define what possibilities are, and the kind of pictures that we have sketched above for disjunctive sentences.

Definition 8 (Possibilities, Pictures) Let $\langle\varphi\rangle^I$ be the meaning of φ.

1. P is a *possibility* for φ iff
   (a) $P \subseteq I$ and for all $i, j \in P$: $\langle i, j\rangle \in \langle\varphi\rangle^I$
   (b) There is no $P' \subseteq I$: $P \subset P'$ and P' satisfies (
2. $\pi[\varphi]$ is a *picture* of φ iff $\pi[\varphi]$ is the set of possibilities for φ

So, a possibility for a sentence is a largest set of indices such that all of them are related to each other in the meaning of the sentence. In the picture of a sentence, all possibilities for the sentence are collected.

Question 5 Do we always get the picture that we want or expect? This is one of the not too many questions asked here to which I know a partial answer: No. Sometimes you get poorer pictures than you would expect, i.e., a smaller class of possibilities than you would have hoped for. Try to find examples yourself! (Of course, I want to explain them away.)

In Figure 1 we have drawn an actual picture of the meaning of $(p \vee q)$, relative to indices which are suitable for the sentence, i.e., indices which just assign a value to p and q. The dots in the picture correspond to such indices, and tell you which value the index gives to p and q, in that order. An arrow between two indices corresponds to the pair of indices being in the meaning of the sentence. The two ovals correspond to the two possibilities for $(p \vee q)$. The index shown in white, is excluded by the meaning of the sentence. .

If we were to draw a picture of the meaning of $!(p \vee q)$ as well, the indices 10 and 11 would be connected to each other as well, and we would end up with a single possibility, including the indices 11, 10, and 01.

Figure 2 shows a picture of $?p$, i.e., of $(p \vee \neg p)$. In this case there are two possibilities as well, but they do not overlap, and no indices are excluded. If we were to draw a picture of $!?p$, all indices would be connected to each other, which means that we end up with a single possibility covering all four indices. This would also be the picture of $\top$. A picture of $\neg ?p$ would

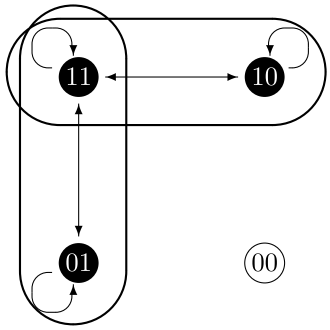

Figure 1: Picture of meaning $(p \vee q)$

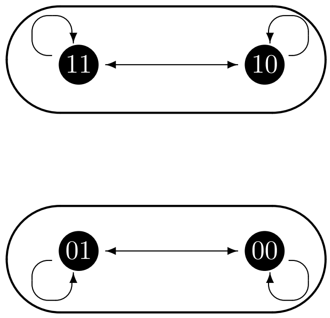

Figure 2: Picture of meaning $?p$

only show the four indices, all in white, and no arrows linking any index to another one, nor to itself. According to the definition $\pi[!?p] = \{\emptyset\} = \pi[\bot]$.

Given the way possibilities are defined, every picture contains at least one possibility. Pictures of inquisitive sentences contain more than one possibility, pictures of informative sentences are such that the union of the possibilities it contains is neither the empty set, nor the set of all indices.

Fact 6 (Informative and Inquisitive Pictures)

1. For all φ: $|\pi[\varphi]| \geq 1$
2. φ is inquisitive iff $|\pi[\varphi]| > 1$.
3. φ is informative iff $\emptyset \neq \bigcup \pi[\varphi] \neq I$.

Being defined as non-inquisitive sentences, pictures of assertions always contain a single possibility, which in case of an informative assertion is neither the empty set nor the set of all indices.

Being defined as non-informative sentences, pictures of questions either contain only the empty set, or the union of the possibilities in the picture equals the set of all indices. A picture of an inquisitive question contains at least two possibilities, and the union of all possibilities in the picture of a question equals the set of all indices.

A picture of a hybrid sentence contains at least two possibilities, and the union of all possibilities in the picture does not equal the set of all indices.

Question 6 What would be the consequences for all definitions and facts if we were to change the definition of possibilities in such a way that we require them to be non-empty?

4.5 Dispensability of Negation

We are on our way to discuss conditional questions, but before we turn to that we have a first look at implication via the issue of the dispensability of negation. We want to show the following:

Fact 7 (Dispensability of Negation) $\neg\varphi \Leftrightarrow (\varphi \to \bot)$

This is a less easy affair than the dispensability of ? and !. The definition of the semantics tells us that $\langle i, j\rangle \models (\varphi \to \bot)$ iff for all $\iota \in \{i, j\}^2$: if $\iota \models \varphi$, then $\iota \models \bot$. Since no $\iota \models \bot$, this means that $\langle i, j\rangle \models (\varphi \to \bot)$ iff $\langle i, j\rangle \not\models \varphi$ and $\langle j, i\rangle \not\models \varphi$ and $\langle i, i\rangle \not\models \varphi$ and $\langle j, j\rangle \not\models \varphi$. The definition of $\langle i, j\rangle \models \neg\varphi$ only mentions the last two conjuncts.

Now, if we would know in general that if $\langle i, i\rangle \not\models \varphi$ and $\langle j, j\rangle \not\models \varphi$, then $\langle i, j\rangle \not\models \varphi$, then we would be done. We will prove something slightly stronger:

Fact 8 (Symmetry and Reflexive Closure)

1. For all i and j: if $\langle i, j\rangle \models \varphi$, then $\langle j, i\rangle \models \varphi$
2. For all i and j: if $\langle i, j\rangle \models \varphi$, then $\langle i, i\rangle \models \varphi$ and $\langle j, j\rangle \models \varphi$

Question 7 I am very fond of this fact. In previous (dynamic) versions of this paper, I always started out stipulating that my 'models' (information states) were relations having this property. Now, I show that it follows from the way the semantics is stated. This is not (yet) a question, but a remark. And here follows another one. The meanings bear resemblances with Kripke models. You can say that the semantics is stated relative to very tiny Kripke models, and the meanings are constructed from them. Now, finally, the question: Can we find out something about the relation between modal logic and what we have here? (The paper by Nelken and Shan mentioned in an earlier question, might be a good starting point to look into this.)

We prove Fact 8 by induction, where we only need to take into consideration the semantic clauses for atomic sentences, $\bot$, $\to$, $\wedge$, and $\vee$ since the other clauses are dispensable. Well, for negation we hope to show that. But if we can prove this fact, we have done so.

First symmetry. For the atomic clause this is trivial, we even have that $\langle i,j\rangle \models p$ iff $\langle j,i\rangle \models p$. And since no $\langle i,j\rangle \models \bot$, symmetry is trivial in this case as well. Also for implication things are rather easy. Since $\langle i,j\rangle \models (\varphi \to \psi)$ iff for all $\iota \in \{i,j\}^2$: if $\iota \models \varphi$, then $\iota \models \psi$, and $\{i,j\}^2$ is the same whether we consider $\langle i,j\rangle \models (\varphi \to \psi)$ or $\langle j,i\rangle \models (\varphi \to \psi)$, symmetrie cannot fail to hold. For $\wedge$ and $\vee$, we need the induction step. If we may assume that $\langle i,j\rangle \models \varphi$ iff $\langle j,i\rangle \models \varphi$, and the same for ψ, then given the way conjunction and disjunction are defined, we can also be sure that $\langle i,j\rangle \models (\varphi \wedge \psi)$ iff $\langle j,i\rangle \models (\varphi \wedge \psi)$. And similarly for disjunction.

Next reflexive closure. The atomic case is trivial. We have that $\langle i,j\rangle \models p$ iff $i(p) = 1$ and $j(p) = 1$; and $i(p) = 1$ iff $\langle i,i\rangle \models p$, and $j(p) = 1$ iff $\langle j,j\rangle \models p$. It cannot fail to be the case that $\langle i,j\rangle \models p$ iff $\langle i,i\rangle \models p$ and $\langle j,j\rangle \models p$. The case of $\bot$ is equally trivial.

The semantic clause for implication tells us that $\langle i,j\rangle \models (\varphi \to \psi)$ iff for all $\iota \in \{i,j\}^2$: if $\iota \models \varphi$, then $\iota \models \psi$. Both $\langle i,i\rangle \in \{i,j\}^2$ and $\langle i,i\rangle \in \{i,j\}^2$. Hence, $\langle i,j\rangle \models (\varphi \to \psi)$ implies that if $\langle i,i\rangle \models \varphi$, then $\langle i,i\rangle \models \psi$, and the same for $\langle j,j\rangle$. By definition: if $\langle i,i\rangle \models \varphi$, then $\langle i,i\rangle \models \psi$ iff $\langle i,i\rangle \models (\varphi \to \psi)$, and the same for $\langle j,j\rangle$. We are done with implication. If $\langle i,j\rangle \models (\varphi \to \psi)$, then $\langle i,i\rangle \models (\varphi \to \psi)$ and $\langle j,j\rangle \models (\varphi \to \psi)$.

Next conjunction, where we have to use induction. We have to prove that: if $\langle i,j\rangle \models (\varphi \wedge \psi)$, then $\langle i,i\rangle \models (\varphi \wedge \psi)$ and $\langle j,j\rangle \models (\varphi \wedge \psi)$. Suppose this was not the case, i.e., $\langle i,j\rangle \models (\varphi \wedge \psi)$; and $\langle i,i\rangle \not\models (\varphi \wedge \psi)$ or $\langle j,j\rangle \not\models (\varphi \wedge \psi)$. By definition, $\langle i,i\rangle \not\models (\varphi \wedge \psi)$ iff $\langle i,i\rangle \not\models \varphi$ or $\langle i,i\rangle \not\models \psi$, and similarly for

$\langle j,j\rangle$. Hence, we have that: $\langle i,i\rangle \not\models (\varphi \wedge \psi)$ or $\langle j,j\rangle \not\models (\varphi \wedge \psi)$ iff $\langle i,i\rangle \not\models \varphi$ or $\langle i,i\rangle \not\models \psi$ or $\langle j,j\rangle \not\models \varphi$ or $\langle j,j\rangle \not\models \psi$. By rearranging the disjuncts: $\langle i,i\rangle \not\models (\varphi \wedge \psi)$ or $\langle j,j\rangle \not\models (\varphi \wedge \psi)$ iff $\langle i,i\rangle \not\models \varphi$ or $\langle j,j\rangle \not\models \varphi$ or $\langle i,i\rangle \not\models \psi$ or or $\langle j,j\rangle \not\models \psi$. Here we have to use the induction step, which tells us that: if $\langle i,i\rangle \not\models \varphi$ or $\langle j,j\rangle \not\models \varphi$, then $\langle i,j\rangle \not\models \varphi$; and similarly for ψ. So, where we are now is that: if $\langle i,i\rangle \not\models (\varphi \wedge \psi)$ or $\langle j,j\rangle \not\models (\varphi \wedge \psi)$, then $\langle i,j\rangle \not\models \varphi$ or $\langle i,j\rangle \not\models \psi$. By the definition of conjunction this means that $\langle i,j\rangle \not\models (\varphi \wedge \psi)$. But this contradicts the assumption, part of which was that: $\langle i,j\rangle \models (\varphi \wedge \psi)$. Hence, we have shown that: if $\langle i,j\rangle \models (\varphi \wedge \psi)$, then $\langle i,i\rangle \models (\varphi \wedge \psi)$ and $\langle j,j\rangle \models (\varphi \wedge \psi)$.

Finally disjunction, where we have to use induction as well. We have to prove that If $\langle i,j\rangle \models (\varphi \vee \psi)$, then $\langle i,i\rangle \models (\varphi \vee \psi)$ and $\langle j,j\rangle \models (\varphi \vee \psi)$. Suppose this was not the case, i.e., $\langle i,j\rangle \models (\varphi \vee \psi)$; and $\langle i,i\rangle \not\models (\varphi \vee \psi)$ or $\langle j,j\rangle \not\models (\varphi \vee \psi)$. By definition, $\langle i,i\rangle \not\models (\varphi \vee \psi)$ iff $\langle i,i\rangle \not\models \varphi$ and $\langle i,i\rangle \not\models \psi$, and similarly for $\langle j,j\rangle$. So, we would have that $\langle i,j\rangle \models (\varphi \vee \psi)$ and: $\langle i,i\rangle \not\models \varphi$ and $\langle i,i\rangle \not\models \psi$, or $\langle j,j\rangle \not\models \varphi$ and $\langle j,j\rangle \not\models \psi$. This would imply that: $\langle i,i\rangle \not\models \varphi$ or $\langle j,j\rangle \not\models \varphi$; and $\langle i,i\rangle \not\models \psi$ or $\langle j,j\rangle \not\models \psi$. Here we need the induction step, i.e., that we may assume that: if $\langle i,j\rangle \models \varphi$ then $\langle i,i\rangle \models \varphi$ and $\langle j,j\rangle \models \varphi$, and the same for ψ. This means that $\langle i,i\rangle \not\models \varphi$ or $\langle j,j\rangle \not\models \varphi$ implies that $\langle i,j\rangle \not\models \varphi$, and similarly, that $\langle i,i\rangle \not\models \psi$ or $\langle j,j\rangle \not\models \psi$ implies that $\langle i,j\rangle \not\models \psi$. But by the definition of disjunction, if $\langle i,j\rangle \not\models \varphi$ and $\langle i,j\rangle \not\models \psi$, then $\langle i,j\rangle \not\models (\varphi \vee \psi)$. But this contradicts our assumption, part of which was that $\langle i,j\rangle \models (\varphi \vee \psi)$. Hence, we have shown that: if $\langle i,j\rangle \models (\varphi \vee \psi)$, then $\langle i,i\rangle \models (\varphi \vee \psi)$ and $\langle j,j\rangle \models (\varphi \vee \psi)$.□

Question 8 This is a proof by brute force. Is there a more elegant way to show this?

Fact 8 shows a couple of things. We have proved that: for all i and j: if $\langle i,j\rangle \models \varphi$, then $\langle i,i\rangle \models \varphi$ and $\langle j,j\rangle \models \varphi$. This entails that: for all i and j: if $\langle i,j\rangle \models \varphi$, then $\langle i,i\rangle \models \varphi$ or $\langle j,j\rangle \models \varphi$. The latter was seen to be necessary to prove the dispensability of negation.

But we have also shown a quite general property of the meanings of sentences $\varphi \in L$. We have proved that for all φ: $\langle \varphi \rangle^I$ is a symmetric relation on I, closed under reflexivity. I..e., that $\langle i,j\rangle \in \langle \varphi \rangle^I$ iff $\langle j,i\rangle \in \langle \varphi \rangle^I$, and if $\langle i,j\rangle \in \langle \varphi \rangle^I$, then $\langle i,i\rangle \in \langle \varphi \rangle^I$ and $\langle j,j\rangle \in \langle \varphi \rangle^I$. And this boils down to the following:

Fact 9 (Symmetry and Reflexivity) For all $\varphi \in L$:

> The meaning of φ, $\langle\varphi\rangle^I$ is a symmetric and reflexive relation on a subset if I.

Question 9 What precisely does this tell us about the properties of pictures?

4.6 Conditional Questions, and More

Let us consider the simplest example of a conditional question: $(p \to ?q)$. The semantics is as follows:

$\langle i, j\rangle \models (p \to ?q)$ iff for all $\iota \in \{i, j\}^2$: if $\iota \models p$, then $\iota \models ?q$

We have that $?q$ is a question, and since questions are not informative, it holds for all $?\varphi$ that for all i: $\langle i, i\rangle \models ?\varphi$. That means that among $\iota \in \{i, j\}^2$ we only need to consider $\langle i, j\rangle$. (We can also forget about $\langle j, i\rangle$ because of symmetry.) Hence we have:

$\langle i, j\rangle \models (p \to ?q)$ iff if $\langle i, j\rangle \models p$, then $\langle i, j\rangle \models ?q$

Note that this simplification holds in all cases where the consequent of a conditional is a question. This boils down to:

$\langle i, j\rangle \models (p \to ?q)$ iff if $i(p) = j(p) = 1$, then $i(q) = j(q)$

We only have that $\langle i, j\rangle \not\models (p \to ?q)$, if $i(p) = j(p) = 1$ and $i(q) \neq j(q)$. The meaning of this conditional question is depicted in Figure 3.

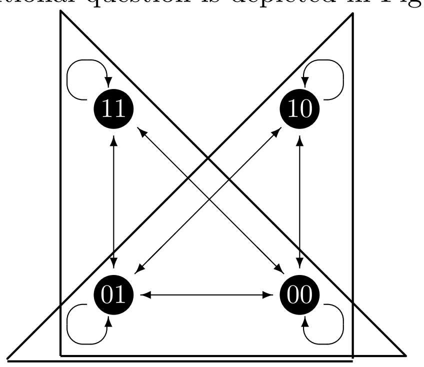

Figure 3: Picture of meaning $(p \to ?q)$

The picture shows that $(p \to ?q)$ is an inquisitive question, it gives rise to two possibilities: the possibility that $(p \to q)$ and the possibility that

$(p \to \neg q)$. Actually, as the picture shows, the conditional question $(p \to ?q)$ is equivalent with the disjunction $((p \to q) \vee (p \to \neg q))$. This is an instance of the more general fact: (which I owe to Salvador Mascarenhas, I think)

Fact 10 $(!\varphi \to (\psi \vee \chi)) \Leftrightarrow ((!\varphi \to \psi) \vee (!\varphi \to \chi))$

Let us have a quick look at the interpretation of assertive conditionals as well. Consider the simplest example $(p \to q)$. The semantics tells us:

$\langle i, j \rangle \models (p \to q)$ iff for all $\iota \in \{i, j\}^2$: if $\iota \models p$, then $\iota \models q$.

Atomic formulas are assertions, they are not inquisitive. That means that among $\iota \in \{i, j\}^2$, we only have to consider $\langle i, i \rangle$ and $\langle j, j \rangle$. Because not being inquisitive implies that if $\langle i, i \rangle \models \varphi$ and $\langle j, j \rangle \models \varphi$, then $\langle i, j \rangle \models \varphi$.

$\langle i, j \rangle \models (p \to q)$ iff if $\langle i, i \rangle \models p$, then $\langle i, i \rangle \models q$ and if $\langle j, j \rangle \models p$, then $\langle j, j \rangle \models q$.

And this is the same as:

$\langle i, j \rangle \models (p \to q)$ iff $\langle i, i \rangle \models (p \to q)$ and $\langle j, j \rangle \models (p \to q)$

This tells us also that $(p \to q)$ is not inquisitive, it is a plain assertion. If we would draw the picture, we would just get the triangle on the left from Figure 3, the index 10 would turn white and would have no arrows to or from any of the four indices in the picture.

Note that the story we have told for $(p \to q)$, holds quite generally for $(\varphi \to \psi)$, as long as the consequent ψ is an assertion, and no matter what the nature of the antecedent φ is. As long as the consequent is an assertion, only $\langle i, i \rangle$ and $\langle j, j \rangle$ have to be taken into consideration. That means in particular that if the antecedent φ is a question, in which case for all i: $\langle i, i \rangle \models \varphi$, the antecedent plays no role: $(?\varphi \to !\psi)$ is equivalent with $!\psi$. This follows from the more general fact:

Fact 11 $(\varphi \to !\psi) \Leftrightarrow (!\varphi \to !\psi)$

In case the consequent of an implication is an assertion, only the informative content of the antecedent plays a role.

In case the consequent is inquisitive, the story is different of course. We saw that already for conditional questions, but it also holds for hybrid conditionals, such as $(p \to (q \vee r))$. According to Fact 10 this is equivalent with $((p \to q) \vee (p \to r))$, which is a hybrid which gives rise to the two posasibilities corresponding with the two disjuncts. So it does not mean the same as $(p \to !(q \vee r))$, which is an assertion giving rise to a single possibility.

Question 10 Good thing to do here: compare with Velissaratou's analysis, which was the starting point for my project, and with a forthcoming paper by Isaacs and Rawlins with stimulated me to to return to this stuff.

If the antecedent is inquisitive as well, as in $((p \vee q) \rightarrow ?r)$, we get yet another effect. This is equivalent with $((p \rightarrow ?r) \wedge (q \rightarrow ?r)$, a conjunction of two conditional questions. This an instance of the following more general fact:

Fact 12 $((\varphi \vee \psi) \rightarrow \chi) \Leftrightarrow ((\varphi \rightarrow \chi) \wedge (\psi \rightarrow \chi))$

We didn't discuss conjunction yet, apart from noting that the meaning of a conjunction is the intersection of the meanings of the conjuncts. In case both conjuncts are assertions, we get completely standard results of course. In Figure 4 we depict the meaning of a simple conjunction of two questions. For the conjunction of questions $(?p \wedge ?q)$ we get four possibilities, where

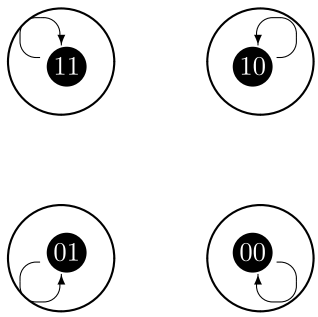

Figure 4: Picture of meaning $(?p \wedge ?q)$

each possibility corresponds with a conjunction of one of the two possibilities for $?p$ and one of the two possibilities for $?q$.

If we now return to $((p \vee q) \rightarrow ?r)$, which we saw to be equivalent with $((p \rightarrow ?r) \wedge (q \rightarrow ?r)$, which is also a conjunction of two polar conditional questions, we also end up with four possibilities corresponding to:

1. $((p \rightarrow r) \wedge (q \rightarrow r)) \Leftrightarrow ((p \vee q) \rightarrow r)$
2. $((p \rightarrow \neg r) \wedge (q \rightarrow \neg r)) \Leftrightarrow ((p \vee q) \rightarrow \neg r)$

3. $((p \to \neg r) \wedge (q \to r))$

4. $((p \to r) \wedge (q \to \neg r))$

In contrast, $(!(p \vee q) \to ?r)$ only gives rise to the first two of these possibilities.

From this example it is a small step to funny questions like: $(?p \to ?q)$. This is equivalent with $((p \to ?q) \wedge (\neg p \to ?q))$. Again a conjunction of two conditional questions, leading to the following four possibilities:

1. $((p \to q) \wedge (\neg p \to q)) \Leftrightarrow q$

2. $((p \to \neg q) \wedge (\neg p \to \neg q)) \Leftrightarrow \neg q$

3. $((p \to q) \wedge (\neg p \to \neg q))$

4. $((p \to \neg q) \wedge (\neg p \to q))$

And this corresponds more or less with: Given an answer to $?p$, what is the answer to $?q$? A picture of its meaning, not explicitly showing the four alternatives, is given in Figure 5.

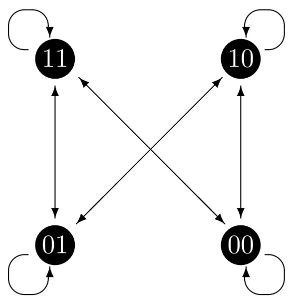

Figure 5: Meaning $?p \to ?q$

The picture of $?p \to ?q$ has the peculiar property that there is a lot of overlap between the possibilities: no possibility contains indices which are unique for that possibility. Every index in a possibility is shared with some other possibility.

This peculiar feature also occurs with disjunctions of question. See Figure 6. As the picture shows, $(?p \vee ?q)$ gives rise to four possibilities, each overlapping with two others, which correspond to $p, \neg p, q, \neg q$. The question can be called a choice question. An answer to any of the two questions, positive or negative, suffices. An example that better illustrates the choice

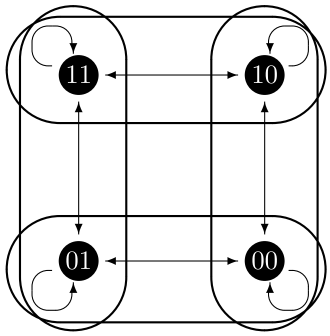

Figure 6: Picture of meaning $(?p \vee ?q)$

nature of disjunctions of questions is $((?p \wedge ?q) \vee (?p \wedge ?r) \vee (?q \wedge ?r))$. It is a question which leaves you the choice of answering two of the three questions $?p$, $?q$ and $?r$. Something I often use in making exams.

One final example, and of a more natural kind of question, is $?(p \vee q)$, which might be called an alternative question. Its meaning is depicted in Figure 7. As the picture shows, there are three possibilities, corresponding

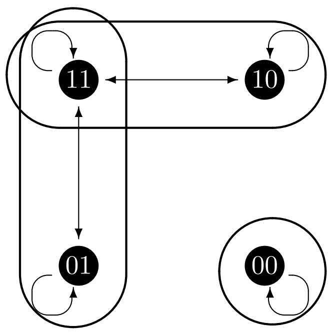

Figure 7: Picture of meaning $?(p \vee q)$

to $p, q, \neg(p \vee q)$. Such alternative questions in the logical language may be taken to correspond to a natural language question like “Do you want coffee, or tea?”, where besides for the answers “Coffee, please”, and “Tea please”, it leaves room for “No, thank you.”

Remember the story about the theme and rheme of a sentence. The alternative question $?(p \vee q)$ is the theme of the disjunction $(p \vee q)$. Of course, $?(p \vee q)$ is the same as $((p \vee q) \vee \neg(p \vee q))$. As compared to $(p \vee q)$ a third possibility is added in which the indices are collected which are excluded by $(p \vee q)$. This is the general pattern underlying arriving from a sentence φ at its theme $?\varphi$.

5 Inquisitive Logic

5.1 Inquisitive Entailment

As any decent logical semantics, inquisitive semantics comes with a corresponding notion of entailment. It is defined quite standardly:

Definition 9 (Inquisitive Entailment)
$\varphi_1, \ldots, \varphi_n \models \psi$ iff for all $\iota \in I^2$: if $\iota \models \varphi_1$ and $\ldots$ and $\iota \models \varphi_n$, then $\iota \models \psi$

And as is to be expected entailment corresponds to meaning inclusion, and we can also characterize entailment in terms of pictures of meaning. I only state the single premisse case.

Fact 13 (Inquisitive Entailment, Meanings and Pictures)

1. $\varphi \models \psi$ iff $\langle \varphi \rangle^I \subseteq \langle \psi \rangle^I$
2. $\varphi \models \psi$ iff every possibility in the picture of φ is included in some possibility in the picture of ψ

I list a rather arbitrary list of entailment facts.

Fact 14 (Some Inquisitive Entailments and Validities)

1. $\varphi \models\, !\varphi$
2. $\varphi \models\, ?\varphi$, $\neg\varphi \models\, ?\varphi$
3. If $\varphi \models \psi$ and $\psi \models \chi$, then $\varphi \models \chi$
4. $\varphi \models \psi$ iff $\models (\varphi \rightarrow \psi)$
5. $(\varphi \wedge \psi) \models \varphi$
6. $\varphi \models (\varphi \vee \psi)$
7. $\not\models (p \vee \neg p)$,

8. $\models !(\varphi \vee \neg\varphi)$

9. $\varphi \models (\psi \rightarrow \varphi)$

10. $(\varphi \vee \psi) \models \neg(\neg\varphi \wedge \neg\psi)$

11. $(\varphi \vee \psi) \models (\neg\varphi \rightarrow \psi)$

12. $(\varphi \rightarrow \psi) \models \neg(\varphi \wedge \neg\psi)$

Question 11 Can you add some characteristic cases to this small list?

5.2 Licensing

The notion of licensing is intended to characterize a logical notion of relatedness of one sentence to another. Of course, entailment also characterizes a notion of relatedness, but not one that deals with relatedness in terms of dialogue coherence. And it is the latter that we are interested in here.

For the moment, we look at just a single move in a dialogue. A sentence has been uttered, wich we call the *stimulus*, and the participant in the dialogue who uttered it we call the stimulator, and then we consider the *response* to that by another participant in the dialogue, the responder. The notion of licensing is to judge whether the response is strictly related to the stimulus.

A typical example of a dialogue coherence relation is of course when the stimulus is a question, and the response a partial answer to that question. Partial answerhood to a question is a relation between sentences that licensing should certainly cover.

It is also generally acknowledged that to replace a question by a subquestion is a significant move in a dialogue. So licensing should cover that as well. Further, we shall also allow for critical responses, such as denying the stimulus, or expressing doubt about it.[9]

The following definition is intended to cover these cases. Licensing is defined in terms of pictures of meaning (and, actually, I know no other way of doing it).

[9]The notion of compliance in the Tbilisi paper as such does not take critical responses into account, but as I remark there, it could easily be adapted to do so.

Definition 10 (Licensing) Let φ and ψ be sentences of L.

> A stimulus ψ *licenses* a response φ, $\psi \propto \varphi$ iff every possibility in the picture of φ is the union of a subset of the set of possibilities in the picture of $?\psi$.

Question 12 In Christopher Potts' calculator for Inquisitive Semantics (see Link in Blackboard) you are asked not to enter more than 4 atomic formulas, otherwise the machinery won't run. A similar device programmed by our PhD student Tikitu de Jager meets the same problem. What is calculated in both cases are the possiblities that a formula gives rise to, according to definition ??. That definition does not give rise to an efficient algorithm.

This what Tikitu wrote about it to Chris in an e-mail: "I had a bit of a look around when I was writing my system, and it seems that finding the maximal clique in a graph (that is, finding the largest alternative [possibility]) is NP-complete. So unless there's something clever we can do based on what we know about the updates (transform old alternatives into new alternatives; I couldn't think of anything), there doesn't seem much chance of an efficient solution."

Now, apart from giving rise to nice pictures of meanings, the *logical* purpose of defining the notion of possibilities is to define the notion of licensing (relatedness) in terms of them. One could ask: Is there an alternative way to define licensing that does not use the notion of possibilities?

The alternative I am thinking of is the following. Take the set of *indifference classes* instead of the set of possibilities. An indifference class for an index $i \in I$ in the meaning $\langle\varphi\rangle^I$ is the set $\{j \in I \mid \langle i, j\rangle \in \langle\varphi\rangle^I\}$. Collect those for all $i \in I$, and call that the set of indifference classes in the meaning of φ. Then restate the definition of licensing relative to sets of indifference classes instead of sets of possibilities: every indifference class in the meaning of φ is the union of a subset of the set of indifference classes for ψ (or $?\psi$).

The question is then: do the old and the new definition have the same effect? Another question is: I am assuming that calculating indifference classes does not have the same complexity as calculating possibilities, is that correct? Finally, assuming positive answers to the two earlier questions, one could also ask whether in terms of indifference classes one could define possibilities in an easier way.

A first thing to note is that licensing does not look at the relation between the response φ and the stimulus ψ as such, but takes the theme $?\psi$ of ψ, the question behind the stimulus, to be what the response should be related to.

This immediately delivers that critical responses are covered. An assertion licenses its negation, and also the corresponding yes/no question, next to 'repeating' the assertion, or remaining 'silent'.

Fact 15 (Licensing by Assertions) Let $!\psi$ be an informative assertion.

$$!\psi \propto \varphi \text{ iff } \varphi \Leftrightarrow !\psi \text{ or } \varphi \Leftrightarrow ?!\psi \text{ or } \varphi \Leftrightarrow \neg\psi \text{ or } \varphi \Leftrightarrow \top$$

Question 13 What happens in case $!\varphi$ is not informative?

The theme of an informative assertion $!\psi$ is the polar question $?!\psi$, in the picture of which we find two mutually exclusive possibilities. The four responses mentioned in the fact exhaust all possibilities, up to equivalence, for φ to be licensed. I will not go into that here and now, but there is a pragmatic story about how precisely to look upon these four responses.

Let us look now at the case where the stimulus is an inquisitive question, and the response an assertion. Since in the picture of an assertions we find only a single possibility, and since $??\psi \Leftrightarrow ?\psi$, Licensing boils down to the following:

Fact 16 (Assertions Licensed by Questions) Let $?\psi$ be an inquisitive question.

> $?\psi \propto !\varphi$ iff the possibility in the picture of φ is a possibility, or the union of some possibilities, in the picture of $?\psi$.

Of course, if the possibility expressed by $!\varphi$ coincides with a single possibility in the picture of $?\psi$, we have a case of complete answerhood. But note that licensing requires that $!\varphi$ should not be overinformative with respect to $?\psi$. E.g., $(p \wedge q)$ is not licensed by $?p$. Another case in point, $\neg p$ is not licensed by $(p \rightarrow ?q)$. The only informative answers $(p \rightarrow ?q)$ licences are $(p \rightarrow q)$ and $(p \rightarrow \neg q)$. Argumentation for overinformative answers not to be licensed comes from the pragmatics.

If $!\varphi$ is an informative assertion that corresponds to the union of some possibilities for $?\psi$, then it is a partial answer. E.g., p is licensed by $(?p \wedge ?q)$, and so are $!(p \vee q)$, and $(p \rightarrow \neg q)$. They exclude some possibility $(?p \wedge ?q)$ gives rise to.

Now we are looking at assertive responses to questions but inquisitive ones can do as well. Such as the alternative question $?(p \vee q)$ and the hybrid disjunction (there it is) $(p \vee q)$ in response to the conjunction of questions $(?p \wedge ?q)$.

Since that's what we started the story with, the Gricean picture of disjunction, let's consider that case in a bit more detail. The hybrid disjunction $(p \vee q)$ can only be licensed by other inquisitive sentences, not necessarily only questions. That is because $(p \vee q)$ gives rise to two overlapping alternatives. If the stimulus is an assertion, the corresponding theme $?!\psi$ gives rise to two non-overlapping alternatives that $(p \vee q)$ cannot meet.

So, with respect to hybrid disjunctions the inquisitive semantics together with the notion of licensing meets Grice's picture precisely. It needs another inquisitive sentences to be contextually related, and each of the possibilities in the disjunction should be a partial answer to that contextual question.

One thing to note is that whenever $(p \vee q)$ is licensed, $!(p \vee q)$ cannot fail to be licensed as well. So, the story is not finished here. We need some pragmatic deliberations to tell us when to choose what, and whether perhaps choosing one over the other gives rise to certain implicatures. (Which I think it does.)

Question 14 Make a list of some (non-)licensing facts, discuss whether you intuitively agree or not with what the definition outputs.

5.3 Classical Licensing

When you do something 'new' it is always important to relate it as well as you can to what went before, to get a precise picture of where the differences are. In our case what went before is partition semantics (and perhaps also the 'conditional semantics' of Velissaratou).

That is one thing. The other thing is that I very much like to argue that the notion of licensing is really 'new' and that it is a basic logical notion, in the sense that it is not reducable to entailment, that inquisitive semantics really comes with a new view of logic.

For partition semantics, classical inquisitiveness, we do have that licensing can be reduced to entailment. So, the idea is that the move to inquisitive semantics, in essence inquisitive disjunction, causes the need for going beyond entailment. (Although, for a logic with just conditional questions, this already holds as well.)

On the semantic side it is easy to characterize the classical case.

Definition 11 (Classical Sentences) Let φ be a sentence of L.

> φ is it classical iff no two possibilities in the picture of φ overlap.

And, of course, we then have the following:

Fact 17 (Classical Pictures are Partitions)

1. φ is classical iff the picture of φ is a partition of a subset of I.
2. φ is classical iff the meaning of φ is an equivalence relation on a subset of I.

Note that I am talking here about partitions on a *subset* of I. That means that the notion not only covers certain questions, but also covers all assertions, and certain hybrids, such as $(p \wedge ?q)$.

I did not really get into that in these notes, but I did mention somewhere that one can show that our inquisitive propositional language is *complete* with respect to the semantics, i.e., that all possible meanings, all relations on subsets of I which are symmetric and closed under reflexivity, can be expressed by a sentence of the language. (Savador Mascarenhas has proved this.) We don't need the full language, $\bot, \wedge, \rightarrow, \vee$ suffices for that, and even only $\neg$ and $\vee$ can manage it.

Now we can ask:

Question 15 Can we characterize a fragment of the full language that is complete with respect to classical meanings? My conjecture is that we get such a fragment, by having $\neg$, $\wedge$ and ? as the only basic operators. (Disjunction, implication defined in terms of that in the classical way, which makes them all assertions.) And if we move to predicate logic, which we should for a good comparison with LoI, we add the universal quantifier. (Existential quantification defined in the classical way turns it into an assertion.) I have no proof of this yet.

We do get alll sorts of things we don't find in LoI, such as negation of a question, but that doesn't really matter, since it is just a contradiction. Also, you don't really find conjunction between questions, and between questions and assertions explicitly in the language of LoI, but since sequences of such sentences are dealt with, I think that in terms of expressiveness it amounts to the same thing. (In terms of the update semantics of LoI, completeness would amount to: every possible information state can be reached by a sequence of updates of the initial state with sentences of the language.)

Then licensing. I think the following holds:

Fact 18 (Classical Licensing) For classical φ and ψ:

$\psi \propto \varphi$ iff $?\psi \models ?\varphi$ (Licensing reducible to entailment)

Note that this is not the same as licensing defined in LoI. It is richer. It not only characterizes partial answerhood for LoI, but also the subquestion relation, and it allows for critical moves.

Question 16 This notion of licensing would not fit the game of interrogation anymore. What sort of game would be characterized by it? Another, and I think much easier, issue is of course to prove (or disprove) the above conjectured fact.

Then the next thing is to show that as soon as we leave the area of classical meanings, this neat characterization of licensing breaks down. (That doesn't yet prove that no reduction of licensing to entailment is possible, only that this one doesn't work anymore, but still.)

The crucial examples are the following:

Fact 19 (Non-Classical Licensing)

1. $(p \rightarrow ?q) \propto (p \rightarrow q)$ and $(p \rightarrow ?q) \propto (p \rightarrow \neg q)$

   $(p \rightarrow ?q) \not\models ?(p \rightarrow q)$ (The other way around!)

2. $?p \not\propto (q \rightarrow ?p)$

   $?p \models (q \rightarrow ?p)$

3. $?p \not\propto (?p \vee ?p)$ (The other way around!)

   $?p \models (?p \vee ?p)$

The first counterexample concerns answerhood. Classical licensing does not deliver the right notion of answerhood for conditional questions. (Note that classical licensing wants $?(p \to ?q)$, but since $(p \to ?q)$ is already a question by itself, putting a question mark in front of it is redundant.)

The second and third counterexample concern the subquestion relation: $?p$ should count as an atomic question, which has no proper subquestions. But since it classically licenses the conditional question $(q \to ?p)$ and the disjunction of questions $(?p \vee ?p)$, classical licensing predicts that $?p$ has these as a kind of subquestions.

Note that especially the second and third case need not really convince you that classical licensing is on the wrong track. One could argue that it is not so bad to predict that $(q \to ?p)$ is licensed by $?p$. You may find that the weaker (entailment is right about that) conditional question is a natural continuation of a dialogue if you are unable to directly answer $?p$.

Well, I agree with that intuition. But licensing is not designed to account for *all* natural continuations of a dialogue. Changing the topic of conversation is also a natural dialogue move, but not one that licensing wants to cover. We want a notion of strict logical relatedness, not the type of weak relatedness that classical licensing predicts in the second example.

Question 17 Try to find a natural situation in which you would say that $(q \to ?p)$ is fine after $?p$. Wouldn't just asking $?q$ be equally fine in the same situation? If so, the logic should not say that $(q \to ?p)$ is licensed by $?p$, because then anything goes.

Also concerning the third example one may have qualms. In particular one might argue that the prediction of non-classical licensing that $(?p \vee ?p) \propto ?p$ is also not particularly convincing.

I have been trying for a long time to change the notion of licensing precisely to get rid of this prediction. The non-success of my many attempts, have convinced me that things should be like this. And that if there is anything wrong with it, it should get a pragmatic explanation, and should not be put in the logic as such.[10]

What makes things not so easy, is that it is not so clear what a disjunction of questions really is. But if you take the choice-question idea seriously, it is rather natural to allow after having been offered a choice between two questions, to just choose one. That's the way licensing looks upon it.

[10] This has changed in the meantime. By adding the notion of homogeneity, these counterexamples are accounted for.

Question 18 There are rather drastic difference between the notion of meaning that inquisitive semantics gives rise to and classical meanings. One such difference is that in a classical semantics, where questions are partitions, the traditional Fregean distinction between sense and reference, intension and extension, remains fully in force, also for questions. The extension of a question in a particular world is *the* proposition that expresses the true and complete answer to the question in that world, the block in the partition to which that world belongs. Following standard patterns, the intension is the *function* that tells you for every world what the extension in that world is.

This feature of the classical partition semantics comes handy, e.g., in accounting for the semantics of questions embedded under a verb like 'to know'. You can analyze 'x knows Q' as to be true in a world w iff x knows the proposition that is the extension of Q in w, i.e., x knows the true answer to the question Q.

This picture breaks down when meanings are sets of possibilities that may overlap. If a world is in the overlap of several possibilities, then there is (or at least there seems to be) no *unique* proposition that corresponds to a true and complete answer in that. Intension as a function from worlds to propositions no longer works.

One thing to do here, is to write a critique of inquisitive semantics from the perspective of Fregean semantics, pointing out what gets lost.

Another thing one might do is to try and see whether one way or the other we can salvage the sense-reference distinction within inquisitive semantics. One may consider this in the semantic format chosen in this paper, or the update/dynamic format I used on the sheets of the last two meetings.

I am primarily thinking of these questions as philosophical questions, but they can also be linked, if only globally, to more empirical issues, like the analysis of sentences with questions that are not partitions (conditional questions, alternative questions) embedded under a verb like 'to know'.

No doubt, there are also other philosophical issues to address with respect to the global inquisitive picture of meaning as related to cooperative information exchange. For example, does it have any impact on Lewis' convention of truthfulness and trust. Such a notion is basically information-oriented, can we give it an inquisitive twist? Perhaps simlar questions could be asked in relation to Davidsonian radical interpretation, where trust in the correspondance of belief also plays a crucial role.

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A Inquisitive Predicate Logic

I clipped this part from an earlier update version of the story, and made minimal changes to make it fit the new simple static semantics. But I'm not fully sure that everything comes out correct.

We present the basics of an extension of our inquisitive semantics to the predicate logical case. We begin with the syntax.

Definition 12 (Inquisitive First Order Syntax) The non-logical vocabulary of L consists of a set of individual constants $\textsc{Con}$, and for $0 \leq n$, a set of n-place predicates $\textsc{Pred}^n$; $\textsc{Var}$ is a denumerable set of variables. By $\textsc{Term}$ we mean $\textsc{Con} \cup \textsc{Var}$.

1. If $P^n \in \textsc{Pred}^n$, and $t_1, \ldots, t_n \in \textsc{Term}$, then $Pt_1 \ldots t_n \in L$.

The clauses 2–9 read the same as in Definition 1.

10. If $x \in \textsc{Var}$, and $\varphi \in L$, then $\exists x\varphi \in L$.
11. If $x \in \textsc{Var}$, and $\varphi \in L$, then $\forall x\varphi \in L$.

As was the case with the connectives, the quantifiers are not interdefinable in the usual way. So, we need both clauses in the syntax.

We assume a standard notion of free and bound variables. By a *sentence* of L, we mean a formula $\varphi \in L$ which has no free occurrences of variables. For easy comparison with *The Logic of Interrogation*, we might add:

$$?\vec{x}\varphi = \vec{\forall x}?!\varphi$$

By $\vec{x}$ we mean a sequence of variables, and by $\vec{\forall x}$ the corresponding sequence of quantifiers.

Next we extend the notion of an index to be able to deal with the extended language.

Definition 13 (Indices and Suitable Pairs)
Let D, the domain, be a non-empty set.

1. An *index* is a function i such that:
   $\forall P^n \in \textsc{Pred}^n$: $i(P^n) \subseteq D^n$; and $\forall t \in \textsc{Term}$: $i(t) \in D$.
2. A pair of indices $\langle i, j \rangle$ is *suitable* iff for all $t \in \textsc{Term}$: $i(t) = j(t)$.

Indices are a combination of an interpretation function for the non-logical vocabulary, and an assignment of values to the variables. Of course variables will be variables and hence their values may vary, but not accross the pairs of indices we take into consideration. You might say that a pair of indices comes with a single assignment. With respect to individual constants, we dictate that they are rigid in a suitable pair.

We add the following little train of notions:

Definition 14 (Assignment)
Let i be a possibility, $\langle i, j\rangle$ a suitable pair of indices.

1. $i[x/d]$ is the index i' which is like i, except for the possible difference that $i'(x) = d$.
2. $\langle i, j\rangle[x/d] = \langle i[x/d], j[x/d]\rangle$.

The inquisitive interpretation of the new syntactic clauses relative to suitable pairs of indices reads as follows.

Definition 15 (Inquisitive Quantification)

1. $\langle i, j\rangle \models Pt_1 \ldots t_n$ iff $\langle i(t_1), \ldots, i(t_n)\rangle \in i(P)$ & $\langle j(t_1), \ldots, j(t_n)\rangle \in j(P)$

The clauses 2–9 in read the same as in Definition 3.

10. $\langle i, j\rangle \models \exists x\varphi$ iff for some $d \in D$: $\langle i, j\rangle[x/d] \models \varphi$
11. $\langle i, j\rangle \models \forall x\varphi$ iff for all $d \in D$: $\langle i, j\rangle[x/d] \models \varphi$

Question 19 What changes do we need to make with respect to the definitions we gave for the propositional case?

Like $(p \vee q)$, $\exists xPx$ is a hybrid sentence. Indices i where $i(P) = \emptyset$ will be eliminated, but furthermore there will be as many possibilities in the picture of $\exists x\varphi$ as there are objects $d \in D$ such that in some index i: $d \in i(P)$. Such a possibility will consist of all the indices i where $d \in i(P)$. The possibilities may overlap. If we take the assertive closure $!\exists xPx$, the difference is that it will not give rise to these different possibilities.

Now, what does this mean? Well, it may need some more deliberation than we have the space for in this Appendix, but I think it means that it provides an answer to the long standing debate about the specific (referential) and non-specific (attributive) use of indefinites.[11] In particular about the issue whether this is a semantic or a pragmatic affair.

We can take $\exists x\varphi$ to correspond to the specific or referential use, where the speaker has a certain individual in mind, only the hearer does not know which, of course, hence she ends up with an issue: which individual is meant?

On the other hand, if $!\exists x\varphi$ is used, it is simply only asserted that there is at least some individual with the property P, it corresponds to the non-specific or attributive use.[12] A general interesting feature about inquisitive semantics is that it shifts the borderline between semantics and pragmatics.

This also holds with respect to questions in combination with existential quantification. If we consider $?\exists xPx$, then it obviously corresponds to a mention-some question, of which it has also been debated whether the distinction between mention-some and mention-all is a semantic or a pragmatic affair.[13]

Apart from the mention-some case, we also get $?!\exists xPx$, which is a yes/no-question, and $\exists x?Px$, which is a choice question (like $(?p \vee ?q)$: pick any object you like, and tell me whether it has the property P or not.

The sentence $\forall xPx$ is an assertion, i.e., unlike in the existential case it is equivalent with $!\forall xPx$, and $?\forall xPx$ is the corresponding yes/no-question.

More interestingly, $\forall x?Px$ corresponds to a who-question, there will be as many possibilities in the picture of $\forall x?Px$, as there are possible denotations of P. It leads to a partition, and we have that $\forall x?Px$ means the same as what $?xPx$ meant in *The Logic of Interrogation*.[14]

Finally, $\forall x(Px \rightarrow ?Qx)$ corresponds to a which-question. And we are back at where our investigations once started. The analysis of which-questions we get is the same as in Velisseratou (2000), and is discussed there in some

[11] I am holding back a bit, because I am looking ahead at dynamic existential quantification, which is not so difficult to implement in the framework as well. If you do that, as has been observed before, the existential quantifier also raises an issue, but this time about the value of the variable quantified over. This, probably fits the specific or referential use even better.

[12] How can we tell which reading is intended? My hypothesis is, accross the board, that we can 'hear' it, that ! and ? correspond to certain intonation patterns or the like.

[13] It would be much harder to defend the position that mention-two, mention-between-5-and-10, etc. is a matter of pragmatics, these of course can be formulated just as easily here.

[14] It would be interesting to look at things like $\forall x?\exists yRxy$ as well, but then, this is just an Appendix.

detail. But now things are embedded in a wider logical context, where we also can provide a characterization of answerhood and subquestionhood for these questions — which give rise to non-overlapping alternatives — in terms of licensing. And this is what I aimed at at the very beginning of the story that, for the time being, ends here.

Question 20 This appendix on the predicate logical case is can be elaborated upon in many ways. You can look at empirical issues that are just hinted at above. You can try to find some logical facts concerning entailment and licensing (or compliance, using the definition from the Tbilisi paper). These things things have hardly been investigated sofar. Also comparisions with, e.g., *The Logic of Interrogation* and Velissaratou's paper could be made.