Inquisitive semantics
—the basics—

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Umass, Amherst, January 25, 2010
Overview

- Motivation
- Basic notions
- Definition of the semantics
- Basic properties
The Traditional Picture

- Meaning = informative content
- Providing information = eliminating possible worlds

- Only captures purely descriptive language use
- Does not reflect the cooperative nature of communication
The Inquisitive Picture

- Propositions as proposals
- A proposal consists of one or more possibilities
- A proposal that consists of several possibilities is inquisitive
A Propositional Language

Basic Ingredients

- Finite set of proposition letters $\mathcal{P}$
- Connectives $\bot$, $\land$, $\lor$, $\rightarrow$

Abbreviations

- Negation: $\neg \varphi := \varphi \rightarrow \bot$
- Non-inquisitive closure: $!\varphi := \neg \neg \varphi$
- Non-informative closure: $?\varphi := \varphi \lor \neg \varphi$
Semantic Notions

Basic Ingredients

- **Index**: function from $\mathcal{P}$ to $\{0, 1\}$
- **Possibility**: set of indices
- **Proposition**: set of alternative possibilities

Notation

- $[\varphi]$: the proposition expressed by $\varphi$
- $|\varphi|$: the truth-set of $\varphi$ (set of indices where $\varphi$ is classically true)

Classical versus Inquisitive

- $\varphi$ is **classical** iff $[\varphi]$ contains exactly one possibility
- $\varphi$ is **inquisitive** iff $[\varphi]$ contains more than one possibility
For any atomic formula $\varphi$: $[\varphi] = \{|\varphi|\}$

Example:
Semantics: negation

\[ \neg \varphi = \left\{ \bigcap_{\alpha \in [\varphi]} \overline{\alpha} \right\} = \{ |\neg \varphi| \} \]

Example, \( \varphi \) classical:

\[
\begin{array}{cc}
11 & 10 \\
01 & 00 \\
\end{array}
\]

\[
\begin{array}{cc}
11 & 10 \\
01 & 00 \\
\end{array}
\]

Example, \( \varphi \) inquisitive:

\[
\begin{array}{cc}
11 & 10 \\
01 & 00 \\
\end{array}
\]

\[
\begin{array}{cc}
11 & 10 \\
01 & 00 \\
\end{array}
\]
Semantics: non-inquisitive closure

\[ ![\varphi] = [\neg\neg\varphi] \]

Example:

\[
\begin{array}{cccc}
11 & 10 \\
01 & 00 \\
\end{array}
\]

\[
\begin{array}{cccc}
11 & 10 \\
01 & 00 \\
\end{array}
\]

\[
\begin{array}{cccc}
11 & 10 \\
01 & 00 \\
\end{array}
\]

\[
\begin{array}{cccc}
11 & 10 \\
01 & 00 \\
\end{array}
\]
Semantics: disjunction (unrestricted)

\[ [\varphi \lor \psi] = [\varphi] \cup [\psi] \]

Examples:

\[
\begin{array}{|c|c|}
\hline
11 & 10 \\
\hline
01 & 00 \\
\hline
\end{array}
\]

\[ p \lor q \]

\[
\begin{array}{|c|c|}
\hline
11 & 10 \\
\hline
01 & 00 \\
\hline
\end{array}
\]

?p (:= p \lor \neg p)
Semantics: conjunction (unrestricted)

\[[\varphi \land \psi] = \{\alpha \cap \beta \mid \alpha \in [\varphi] \text{ and } \beta \in [\psi]\}\]

Example, \(\varphi\) and \(\psi\) classical:

\[
\begin{array}{cc}
11 & 10 \\
01 & 00 \\
\end{array}
\quad
\begin{array}{cc}
11 & 10 \\
01 & 00 \\
\end{array}
\quad
\begin{array}{cc}
11 & 10 \\
01 & 00 \\
\end{array}
\]

\(p\) \quad \(q\) \quad \(p \land q\)

Example, \(\varphi\) and \(\psi\) inquisitive:

\[
\begin{array}{cc}
11 & 10 \\
11 & 10 \\
\end{array}
\]
Semantics: implication (unrestricted)

\[ [\varphi \rightarrow \psi] = \{ \Pi_f \mid f : [\varphi] \rightarrow [\psi] \} \]

where \( \Pi_f = \bigcap_{\alpha \in [\varphi]} (\alpha \Rightarrow f(\alpha)) \)

Examples, classical and inquisitive:

\[
\begin{array}{c|c|c|c|c}
11 & 10 & 01 & 00 & \\
p \rightarrow q & & & & \\
\end{array}
\]

\[
\begin{array}{c|c|c|c|c}
11 & 10 & 01 & 00 & \\
p \rightarrow ?q & & & & \\
\end{array}
\]
Informativeness and Inquisitiveness

- $p \lor q$ is inquisitive: $[p \lor q]$ consists of more than one possibility
- $p \lor q$ is informative: $[p \lor q]$ proposes to eliminate indices

For any formula $\varphi$: $|\varphi| = \bigcup[\varphi]$

$\Rightarrow \bigcup[\varphi]$ captures the informative content of $\varphi$
$\Rightarrow$ classical notion of informative content is preserved.
Questions, Assertions, and Hybrids

- $\varphi$ is a question iff it is inquisitive and not informative
- $\varphi$ is an assertion iff it is informative and not inquisitive
- $\varphi$ is a hybrid iff it is both informative and inquisitive
- $\varphi$ is insignificant iff it is neither informative nor inquisitive
Non-inquisitive closure

- Double negation has the effect of removing inquisitiveness
- For any formula $\varphi$: $[\neg\neg\varphi] = \{\varphi\}$

Therefore, $\neg\neg\varphi$ is abbreviated as $!\varphi$ and called the non-inquisitive closure of $\varphi$
Inquisitiveness and Disjunction

• Classically, \( p \lor \neg p \) is a tautology
• In inquisitive semantics it is not informative, but it is inquisitive
• It is in fact a question, abbreviated as \(?p\)

• Disjunction is, for now, the only source of inquisitiveness in our language

\[ p \lor \neg p \]

\[ \Rightarrow \text{any disjunction-free formula is classical (non-inquisitive)} \]