An inquisitive dynamic epistemic logic

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Outline

Main result

A system that combines the main features of dynamic epistemic logic with those of inquisitive semantics

Roadmap

1. Brief review of DELQ (van Benthem, Miniča, . . .)

2. Brief review of INQB (Ciardelli, Groenendijk, Roelofsen, . . .)

3. An inquisitive epistemic logic, IEL

4. An inquisitive dynamic epistemic logic, IDEL
Dynamic epistemic logic with questions

Language (simplified)

\[ p \mid \neg \varphi \mid \varphi \land \psi \mid \varphi \lor \psi \mid \varphi \rightarrow \psi \mid K_a \varphi \mid [!] \varphi \psi \mid [?] \varphi \psi \]

- \([!] \varphi \psi = \text{‘asserting } \varphi \text{ leads to a state where } \psi \text{ holds’} \]
- \([?] \varphi \psi = \text{‘asking whether } \varphi \text{ leads to a state where } \psi \text{ holds’} \]
Dynamic epistemic logic with questions

Epistemic issue models

\[ M = \langle W, \sim \mathcal{A}, \approx \mathcal{A}, V \rangle \]

- \( \sim \mathcal{A} = \{ \sim a \mid a \in \mathcal{A} \} \)
  
  A set of equivalence relations on \( W \) encoding **epistemic indistinguishability** for each agent

- \( \approx \mathcal{A} = \{ \approx a \mid a \in \mathcal{A} \} \)
  
  A set of equivalence relations on \( W \) encoding the **issues** that have been raised by each agent
Dynamic epistemic logic with questions

Interpretation

• The static fragment of the language is interpreted as usual

• Dynamic speech act operators change the model of evaluation
  • Assertions provide information; they change $\sim \mathcal{A}$
  • Questions raise issues; they change $\approx \mathcal{A}$

• Crucial clauses:
  • $M, w \models [!] \varphi \psi$ iff $M^1\varphi, w \models \psi$
  • $M, w \models [?] \varphi \psi$ iff $M^?\varphi, w \models \psi$
Dynamic epistemic logic with questions

Discussion

- The basic static fragment of the language, and its semantic interpretation, are completely classical

- Questions enter the picture at the level of speech acts

- The basic static language does not contain sentences that are interrogative in any syntactic sense, or inquisitive in any semantic sense
Dynamic epistemic logic with questions

Alternative approach

• Change the semantics of the basic static fragment of the language in such a way that the meaning of a sentence embodies both its informative and its inquisitive content

• Add interrogative sentences, $?\varphi$, to the static language

• The dynamic part of the language can then be simplified. We just need a single general purpose speech act operator:

$$[\varphi]\psi = \text{`uttering } \varphi \text{ leads to a state where } \psi \text{ holds'}$$
Main advantage of the alternative approach

If inquisitiveness enters the picture at the syntactic/semantic level, it becomes possible to deal with embedded questions

(1) John knows who will come to the party. \[ K_a ? x. P x \]
(2) John knows whether Mary will come to the party. \[ K_a ? p \]
(3) If it rains, will Mary still come to the party? \[ p \rightarrow ? q \]

Embedded questions cannot be dealt with straightforwardly in DELQ, because sentences like \( K_a ? q \) and \( p \rightarrow ? q \) are not in \( L_{DELQ} \)
Inquisitive semantics

Language

$p \mid \neg \varphi \mid \varphi \land \psi \mid \varphi \lor \psi \mid \varphi \rightarrow \psi \mid ?\varphi$

- Interrogative sentences, but
- no knowledge operators, and
- no speech act operators
Inquisitive semantics

Models

- Sentences are evaluated relative to information states, i.e., sets of possible worlds
- The central notion is support, rather than truth
- In uttering a sentence $\varphi$, a speaker proposes to update the common ground in such a way that it comes to support $\varphi$
Inquisitive semantics

Support

1. \( s \models p \) iff \( \forall w \in s : w(p) = 1 \)
2. \( s \models \neg \varphi \) iff \( \forall w \in s : \{w\} \models \varphi \)
3. \( s \models \varphi \land \psi \) iff \( s \models \varphi \) and \( s \models \psi \)
4. \( s \models \varphi \lor \psi \) iff \( s \models \varphi \) or \( s \models \psi \)
5. \( s \models \varphi \rightarrow \psi \) iff \( \forall t \subseteq s : \text{if } t \models \varphi \text{ then } t \models \psi \)
6. \( s \models ?\varphi \) iff \( s \models \varphi \) or \( s \models \neg \varphi \)

Propositions and possibilities

• \([\varphi] = \) the set of all states supporting \( \varphi \)
• A possibility for \( \varphi \) is a maximal state supporting \( \varphi \)
Inquisitive semantics

Illustration
The semantics applies deals in a uniform way with declaratives, interrogatives, and embedded interrogatives:
Inquisitive semantics

Informative content
In uttering a sentence $\varphi$, a speaker proposes to eliminate all worlds that are not contained in any state supporting $\varphi$

- $\text{info}(\varphi) = \bigcup[\varphi]$

Informative and inquisitive sentences
- $\varphi$ is informative iff $\text{info}(\varphi) \neq \mathcal{W}$
- $\varphi$ is inquisitive iff $\text{info}(\varphi) \not\models \varphi$
Questions and assertions

- $\varphi$ is a question iff it is non-informative.
- $\varphi$ is an assertion iff it is non-inquisitive.
Inquisitive semantics

Discussion

• **Inquisitiveness** enters the picture at the level of **sentences** and their **semantic content**

• The system deals straightforwardly with **conditional questions**

• It does not deal with **knowledge-wh** ascriptions yet, because $\mathcal{L}_{\text{INQB}}$ does not contain knowledge operators

• The system does not allow us to specify precisely what happens at the **speech act level**

• Thus, integrating inquisitive semantics with dynamic epistemic logic will help both traditions a step further
Inquisitive epistemic logic

Language

\[ p \mid \neg \varphi \mid \varphi \land \psi \mid \varphi \lor \psi \mid \varphi \rightarrow \psi \mid ?\varphi \mid K_a\varphi \]

- Interrogative sentences
- Knowledge operators
- No speech act operators
Inquisitive epistemic logic

States

- Sentences are still evaluated relative to states

- States are now sets of worlds in the canonical model for S5 (or some other epistemic logic)

- As before, the central notion is support, rather than truth

- In uttering a sentence $\varphi$, a speaker proposes to update the common ground in such a way that it comes to support $\varphi$
Inquisitive epistemic logic

Support

1. $s \models p \iff \forall w \in s : V^c(w, p) = 1$
2. $s \models \neg \varphi \iff \forall w \in s : \{w\} \not\models \varphi$
3. $s \models \varphi \land \psi \iff s \models \varphi \text{ and } s \models \psi$
4. $s \models \varphi \lor \psi \iff s \models \varphi \text{ or } s \models \psi$
5. $s \models \varphi \rightarrow \psi \iff \forall t \subseteq s : \text{if } t \models \varphi \text{ then } t \models \psi$
6. $s \models ?\varphi \iff s \models \varphi \text{ or } s \models \neg \varphi$
7. $s \models K_a \varphi \iff \forall w \in s : \sigma_{a, w} \models \varphi$

Propositions

- As before, $[\varphi] = \{s \mid s \models \varphi\}$
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Knowledge ascription

We now have a unified treatment of knowledge-
that and knowledge-
wh ascription:

(4) John knows that Peter will come. \[ K_a p \]
(5) John knows whether Peter will come. \[ K_a ? p \]

A state \( s \) supports \( K_a ? p \) iff for every \( w \in s \),
a’s information state in \( w \) supports either \( p \) or \( \neg p \).
Inquisitive epistemic logic

All the central notions from INQB carry over directly to IEL

- $\text{info}(\varphi) = \bigcup [\varphi]$
- $\varphi$ is informative iff $\text{info}(\varphi) \neq \mathcal{W}$
- $\varphi$ is inquisitive iff $\text{info}(\varphi) \not\models \varphi$

This brings us to the final step: adding a dynamic layer
Inquisitive dynamic epistemic logic

Language

\[ p \mid \lnot \varphi \mid \varphi \land \psi \mid \varphi \lor \psi \mid \varphi \rightarrow \psi \mid ? \varphi \mid K_a \varphi \mid [\varphi]_a \psi \]

- Interrogative sentences
- Knowledge operators
- One speech act operator
  \[ [\varphi]_a \psi = \text{‘an utterance of } \varphi \text{ by } a \text{ leads to a state supporting } \psi \text{’} \]
Discourse contexts

- Sentences will be evaluated relative to a discourse context.
- A discourse context is a pair \(\langle s, T \rangle\), where:
  - \(s\) is a state \(\Rightarrow\) representing the information that has been provided so far.
  - \(T\) is a stack of IEL-propositions \(\Rightarrow\) representing the proposals that have been made so far.
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Changing the discourse context

• Utterances change the discourse context

• $s^{\varphi_a} = \{ w \in s \mid \sigma_{a,w} \subseteq \text{info}(\varphi) \}$

• $T^{\varphi_a} = T + [\varphi]$
Inquisitive dynamic epistemic logic

Support

1. $\langle s, T \rangle \models p \iff \forall w \in s : V^c(p, w) = 1$
2. $\langle s, T \rangle \models \neg \varphi \iff \forall w \in s : \langle \{w\}, T \rangle \not\models \varphi$
3. $\langle s, T \rangle \models \varphi \land \psi \iff \langle s, T \rangle \models \varphi \text{ and } \langle s, T \rangle \models \psi$
4. $\langle s, T \rangle \models \varphi \lor \psi \iff \langle s, T \rangle \models \varphi \text{ or } \langle s, T \rangle \models \psi$
5. $\langle s, T \rangle \models \varphi \rightarrow \psi \iff \forall s' \subseteq s : \text{if } \langle s', T \rangle \models \varphi \text{ then } \langle s', T \rangle \models \psi$
6. $\langle s, T \rangle \models ?\varphi \iff \langle s, T \rangle \models \varphi \text{ or } \langle s, T \rangle \models \neg \varphi$
7. $\langle s, T \rangle \models K_a \varphi \iff \forall w \in s : \langle \sigma_{a,w}, T \rangle \models \varphi$
8. $\langle s, T \rangle \models [\varphi]_a \psi \iff \langle s^{\varphi_a}, T^{\varphi_a} \rangle \models \psi$

Note that the first seven clauses are essentially the same as in IEL
Inquisitive dynamic epistemic logic

Discussion

- IDEL brings together the main features of DEL and INQ

- Main vantage points from the viewpoint of DEL:
  - Inquisitiveness at the level of semantic content
  - Allows for a straightforward account of embedded questions

- Main vantage points from the viewpoint of INQ:
  - Perspicuous representation of the conversational participants’ epistemic states
  - Explicit account of how utterances affect the discourse context
Thank you for your attention

www.illc.uva.nl/inquisitive-semantics