Inquisitive Semantics and Pragmatics[*]
and *The Logic of Interrogation*

Handout

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Introduction

We provide an introduction to inquisitive semantics, but largely we do so by *not* considering inquisitive semantics, but proto-inquisitive semantics, by which we mean the partition semantics given in Groenendijk (1999) for a first-order language with indicatives and interrogatives. Only sentences in the first category can be informative, and only those in the second category can be inquisitive.

In inquisitive semantics not all questions correspond to partitions, which we will motivate by considering conditional questions. Also, in the logical language that the semantics is applied to, no syntactic distinction between indicatives and interrogatives is made. Instead, assertions and questions are characterized semantically. And, not all sentences belong to one of these two categories, there are hybrids, sentences which are both informative and inquisitive.

We will reformulate proto-inquisitive semantics using the concepts and tools from inquisitive semantics, including the logical pragmatical notion of a compliant response from Groenendijk and Roelofsen (2009). This is useful as such, because what results is a conceptually more transparant and a more standard system. Moreover, having the two systems in the same format makes comparison—detecting precisely what the differences are—easier.

Central to the paper is to show that inquisitive semantics offers a new notion of meaning that is richer than that of a classical proposition modeled as a set of possible worlds. Our propositions are modeled as sets of possibilities, which correspond to states in which a sentence is supported. In this way the proposition expressed by a sentence captures both informative and inquisitive content.

Overview

In Section 1 we very briefly motivate and introduce *inquisitive semantics for a propositional language*. In Section 2 we quickly introduce the first order logical language with *indicatives and interrogatives* from Groenendijk (1999) to which we will apply the semantic concepts and tools of inquisitive semantics. That is the main focus of the paper and is dealt with extensively in Section 3 on *proto-inquisitive semantics*. We will see what makes the difference between real and proto-inquisitive semantics. In Section 4 we turn to the logical-pragmatical notion of *compliant responses*. We end in Section 5 by drawing some *conclusions*.

1 Propositional inquisitive semantics

1.1 Conditional questions

A conditional question like (1) can typically be answered by (2) or (3):[1]

(1)	If Alf goes to the party, will Bea go as well?	$p \to ?q$
(2)	Yes, if Alf goes, then Bea will go as well.	$p \to q$
(3)	No, if Alf goes, then Bea will not go.	$p \to \neg q$

There does not seem to be much of a difference with a polar question like (4).

(4)	Will Alf go to the party?	$?p$
(5)	Yes, Alf will go to the party.	p
(6)	No, Alf will not go to the party.	$\neg p$

The questions in (4) and (2) do differ from each other in that the answers in (5) and (6) to (4) mutually exclude each other, whereas the answers in (2) and (3) to (1) do not, as is depicted in Figure 1.
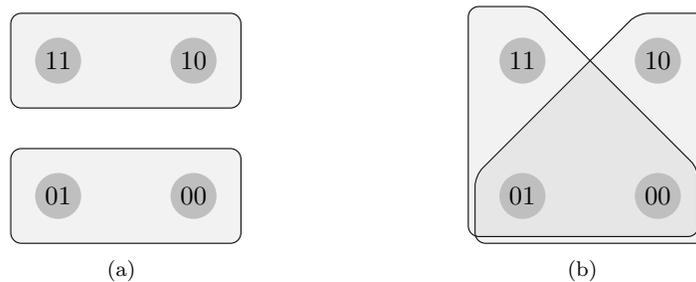

Figure 1: (a) the polar question $?p$ and (b) the conditional question $p \to ?q$

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[1] The first semantics for conditional questions along the lines sketched here can be found in Velissaratou (2000).

This is a problem for a partition semantics for questions, where—by the very definition of what a question is—different answers to a question mutually exclude each other.[2] Inquisitive semantics solves this problem in an easy way.[3]

1.2   Inquisitive semantics in a nutshell

Definition 1 (Language) *Let $\mathcal{P}$ be a finite set of proposition letters. We denote by $\mathcal{L}_\mathcal{P}$ the set of formulas built up from letters in $\mathcal{P}$ and $\perp$ using the binary connectives $\wedge, \vee$ and $\rightarrow$.*

We use of the following abbreviations: $\neg\varphi := \varphi \rightarrow \perp$, $!\varphi := \neg\neg\varphi$ (*non-inquisitive closure*), and $?\varphi := \varphi \vee \neg\varphi$ (*non-informative closure*).

Definition 2 (Worlds and states)

   A world *is a function from $\mathcal{P}$ to $\{0,1\}$. A* state *is a set of worlds.*

We give a support-definition for the language relative to states.

Definition 3 (Support)

   1. $s \models p$ iff $\forall w \in s : w(p) = 1$

   2. $s \models \perp$ iff $s = \emptyset$

   3. $s \models \varphi \wedge \psi$ iff $s \models \varphi$ and $s \models \psi$

   4. $s \models \varphi \vee \psi$ iff $s \models \varphi$ or $s \models \psi$

   5. $s \models \varphi \rightarrow \psi$ iff $\forall t \subseteq s : if\ t \models \varphi\ then\ t \models \psi$

Fact 1 (Support for negation)

   1. $s \models \neg\varphi$ iff $\forall w \in s : w \models \neg\varphi$

   2. $s \models !\varphi$ iff $\forall w \in s : w \models \varphi$

In terms of support, we define the *possibilities* for a sentence φ and the *proposition* expressed by φ. We also define the *truth-set* of φ, which is the meaning that would be associated with φ in a classical setting.

Definition 4 (Truth sets, possibilities, propositions) *Let φ be a formula.*

   1. A possibility *for φ is a maximal state supporting φ, that is, a state that supports φ and is not properly included in any other state supporting φ.*

   2. The proposition *expressed by φ, $[\varphi]$, is the set of possibilities for φ.*

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[2]The classical reference for partition semantics is Groenendijk and Stokhof (1984)

[3]The main references for the basic system of inquisitive semantics sketched here are Ciardelli (2009); Groenendijk and Roelofsen (2009); Ciardelli and Roelofsen (2011). See the webpage `www.illc.uva.nl/inquisitive-semantics` of the inquisitive semantics research project for more publications and up to date information.
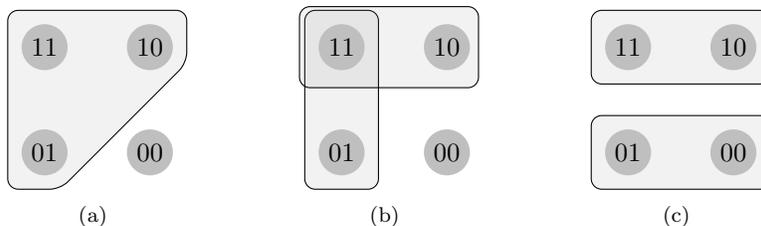
Figure 2: (a) classical picture of $p \vee q$, (b) inquisitive picture of $p \vee q$, and (c) polar question $?p$.

    3. The truth set *of φ, $|\varphi|$, is the set of all worlds w such that $\{w\} \models \varphi$.*

Example 1 (Disjunction) Inquisitive semantics differs from classical semantics in its treatment of disjunction. To see this, consider figures 2(a) and 2(b). In these figures, it is assumed that $\mathcal{P} = \{p, q\}$; world 11 makes both p and q true, world 10 makes p true and q false, etcetera. Figure 2(a) depicts the classical meaning of $p \vee q$: the set of all worlds that make p, q or both p and q true. Figure 2(b) depicts the proposition expressed by $p \vee q$ in inquisitive semantics. It consists of two possibilities. One possibility is made up of all worlds that make p true, and the other of all worlds that make q true.

The inquisitive treatment of disjunction can form the basis for an explanation of the fact that not only an alternative question like (7), but also an indicative disjunction like (8) invites for a response like (9).[4]

(7)   Will ALF or BEA go to the party?

(8)   ALF or BEA will go to the party.

(9)   ALF will go.

After this brief preview of inquisitive semantics, we take a step back, and consider a partition semantics from the perspective of inquisitive semantics. In doing so we will also learn more about inquisitive semantics as such.

2   Indicatives and interrogatives

In proto-inquisitve semantics we are concerned with the semantics for a first-order language with two distinct sentential syntactic categories: indicatives and interrogatives. This is alien to real inquisitive semantics, which does not assume categorical syntactic distinctions among the sentences in the logical language.

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[4]Alternative questions and other disjunctive questions are an intricate affair. For an analysis in inquisitive semantics, see Roelofsen and van Gool (2010); Pruitt and Roelofsen (2011).

2.1 Basic logical language $\mathcal{L}$

The logical language we will investigate is constructed in two steps. We start from a basic language for standard first-order logic and its interpretation.

- We take $\mathcal{L}$ to be a standard language of first-order logic with individual constants L_C and predicate symbols $\mathcal{L}_P$.

- We call models for $\mathcal{L}$ *worlds*. So, a world v is a pair $\langle D, I \rangle$, where D is the domain of v and I is an interpretation function which assigns a denotation to all basic expressions $\alpha \in \mathcal{L}_C \cup \mathcal{L}_P$ relative to D. If I is the interpretation function in v, we denote $I(\alpha)$ by $v(\alpha)$.

- We take $v \models_g \varphi$ to be a standard recursively defined satisfaction relation, which tells us for each $\varphi \in \mathcal{L}$ whether φ is true or not relative to a model v and an assignment function g.

- We call the set of all worlds where a formula φ is true relative to an assignment g, $|\varphi|_g = \{v \mid v \models_g \varphi\}$, the *classical proposition* expressed by φ relative to g. If φ is a *sentence*, a formula in which no free variables occur, we drop reference to an assignment and say that $|\varphi|$ is the *classical proposition* expressed by φ.

Next, we extend the basic language to the full language with two sentential categories: indicatives and interrogatives.

2.2 Proto-inquisitive language

We extend the basic language $\mathcal{L}$ to a proto-inquisitive language language $\mathcal{L}_?$.[5]

Definition 5 (Indicatives, interrogatives, discourses)

1. *If φ is a sentence in $\mathcal{L}$ then $!\varphi$ is an* indicative sentence *in $\mathcal{L}_?$,*

2. *If φ is a formula $\varphi \in \mathcal{L}$, then $?\varphi$ is an* interrogative sentence *in $\mathcal{L}_?$.*

3. *If $\varphi_1, \ldots, \varphi_n$ are sentences in $\mathcal{L}_?$, then $\varphi_1 ; \ldots ; \varphi_n$ is a discourse in $\mathcal{L}_?$.*

- All free occurrences of variables in φ are bound in $?\varphi$.

- We use φ, ψ as meta-variables ranging over sentences and discourses. (We will often say 'sentence' where we more generally mean refer to discourses.)

Example 2 (A discourse)

(10)	$?P(x)$	Who went to the party?
(11)	$!\forall x.(P(x) \leftrightarrow (x = a \lor x = b))$	Only Alf and Bea went to the party.
(12)	$?R(x,y)$	Who danced with whom?

(13)	$!\forall x.\forall y.(x \neq y \rightarrow R(x,y))$	Everyone danced with everyone else.
(14)	$?\exists x.D(x)$	Did anyone get drunk?
(15)	$!\neg\exists x.D(x)$	Nobody got drunk.

2.3 Note on translation

We can translate $\mathcal{L}_?$ into $\mathcal{L}$ in a meaning preserving way, under a proto-inquisitive interpretation for $\mathcal{L}_?$ and a real inquisitive interpretation for $\mathcal{L}$, as follows: (i) $(!\varphi)^\star = !\varphi$; (ii) $(?\varphi)^\star = \forall x_1 \ldots \forall x_n ?!\varphi$, where $x_1, \ldots, x_n$, $n \geq 0$ are the free variables in φ; (iii) $(\varphi_1 ; \ldots ; \varphi_n)^\star = (\varphi_1)^\star \wedge \ldots \wedge (\varphi_n)^\star$.[6]

3 Proto-inquisitive semantics

3.1 The common ground of a discourse

Since the common ground plays an important role in our story, we will be explicit about what we take to be the common ground of a discourse.

We start from a discourse model, and define the initial common ground of a discourse in terms of that.

Definition 6 (Common ground)

- *A* discourse model $\mathbb{D}$ *for* $\mathcal{L}$ *is a pair* $\langle D, I \rangle$, *where D is a non-empty set of objects,* the domain of the discourse, *and I is a* partial *interpretation function for* $\mathcal{L}_C \cup \mathcal{L}_P$ *relative to D.*

- *Let* $\mathbb{D} = \langle D, I \rangle$ *be a discourse model for* $\mathcal{L}$. *The initial common ground of a discourse based on* $\mathbb{D}$, $\omega_{\mathbb{D}}$, *is the set of all worlds (models) v for $\mathcal{L}$ which have D as their domain, and such that $v(\alpha) = I(\alpha)$ for all $\alpha \in \mathcal{L}_C \cup \mathcal{L}_P$.*

If an individual constant or predicate symbol α in the language is in the domain of the interpretation function of the discourse model, then its denotation is the same in all worlds in the common ground, what the denotation of α is belongs to the initial common ground.

Assumption 1 (Fixed Domain) *It follows from the definition of the common ground that we take it for granted that* the domain of discourse is fixed.

Further we make a more specific assumption:

Assumption 2 (Names are rigid) *For all* names *of objects in the domain, i.e., all* individual constants $c \in \mathcal{L}_C$, c *is in the domain of the interpretation function in the discourse model.*

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[6]What we are dealing with here is first-order inquisitive semantics which is not introduced in this paper. See Ciardelli (2010).

The initial common ground is the starting point of a discourse. We take it to be the purpose of a discourse to update the common ground by exchanging information. To update the common ground means to eliminate worlds from it.[7]

3.2 Support in the Common Ground

We give the semantics by a recursive support-definition relative to states.

Definition 7 (States)

A state *is a subset of the set of worlds in the initial common ground ω.*

We call the empty set the fatal state*.*

- Think of a state as the current state of the common ground of a discourse.

The support-definition for $\mathcal{L}_?$ makes use of the truth definition for $\mathcal{L}$:

Definition 8 (Support of a discourse in a state)

1. $s \models {!}\varphi \ $ *iff* $\ s \subseteq |\varphi|$;

2. $s \models {?}\varphi \ $ *iff for all* $v, w \in s$ *and all assignments* $g \colon v \in |\varphi|_g$ *iff* $w \in |\varphi|_g$;

3. $s \models \varphi_1 ; \ldots ; \varphi_n \ $ *iff* $\ s \models \varphi_1$ *and* $\ldots$ *and* $s \models \varphi_n$.

Example 3 The common ground supports the indicative $!\exists x.P(x)$ iff it is part of the common ground that the denotation of P is not empty.

Example 4 The common ground supports the interrogative $?\exists x.P(x)$ iff it is either part of the common ground that the denotation of P is not empty, or that its denotation is empty, the issue whether or not there is an object with property P is fully resolved in the common ground.

Example 5 The common ground supports the interrogative $?P(x)$ iff it is part of the common ground what the denotion of P is, the issue which objects do and which objects do not have property P is fully resolved in the common ground.

Example 6 The common ground supports the interrogative $?R(x, y)$ iff it is part of the common ground what the denotion of R is, the issue which pairs of objects do and which pairs of objects do not stand in the relation R to each other is fully resolved in the common ground.

Definition 9 (Meaning and informative content)

The meaning *of φ is the set of all states that support it.*

The informative content *of φ is the union of all states that support it.*

Notation: meaning *of* $\varphi := ||\varphi||$, informative content *of* $\varphi := \mathit{info}(\varphi) = \bigcup ||\varphi||$.

- Meaning $\neq$ informative content.

3.3 Informativeness and Inquisitiveness

- Read $s \cap \mathsf{info}(\varphi)$ as: the update of s with the informative content of φ.

- Classically, it always holds that $s \cap |\varphi| \models \varphi$.

Definition 10 (Acceptability, informativeness and inquisitiveness)

1. *φ is acceptable in s iff $s \cap \mathbf{info}(\varphi) \neq \emptyset$.*

2. *φ is informative in s iff $s \cap \mathbf{info}(\varphi) \neq s$.*

3. *φ is inquisitive in s iff $s \cap \mathbf{info}(\varphi) \not\models \varphi$.*

4. *φ is inquisitive iff φ is inquisitive in ω.* Similarly for the other two notions.

- For an inquisitive sentence it is not sufficient to add its informative content (if it has any) to the current state to get at a state that supports it.

- An inquisitive sentence raises an issue. Additional information that resolves the issue is needed to get at a state that supports it.

Fact 2 (Inquisitiveness and acceptability)

If φ is unacceptable, then φ is informative.

If φ is inquisitive, then φ is acceptable.

3.4 Assertions and Questions

Definition 11 (Five semantic categories)

1. *φ is a contradiction iff φ is not acceptable.*

2. *φ is an assertion iff φ is not inquisitive.*

3. *φ is a question iff φ is not informative.*

4. *φ is hybrid iff φ is informative and inquisitive.*

5. *φ is a tautology iff φ is neither informative nor inquisitive.*

A contradiction is informative; an assertion need not be informative; a question need not be inquisitive; a tautology is both a question and an assertion.[8]

Fact 3 (Assertions and Questions)

$!\varphi$ is an assertion, and $?\varphi$ is a question.

Below we use **Proto-Fact** to label facts that are peculiar to the restricted setting of proto-inquisitive semantics. Plain **Facts** also apply to the propositional inquisitive semantics presented in Section 1, but not always to the first-order case. This will be noted at the most crucial point.

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[8]If you find these consequences of the definition of the five categories counterintuitive, see Groenendijk and Roelofsen (2009) for an alternative definition which prevents them.

Proto-Fact 1 (Hybridness) *There are no hybrid single sentences in $\mathcal{L}_?$, only a discourse consisting of at least one assertion and at least one question can be hybrid.*

- In real inquisitive semantics, a plain disjunction like $p \vee q$ is a hybrid (single) sentence.

3.5 Entailment and equivalence

Definition 12 (Entailment and Equivalence)

- $\varphi \models \psi$ *iff every state that supports φ, supports ψ as well.*

- $\varphi \equiv \psi$ *iff φ and ψ are supported by the same states.*

Fact 4 (Assertoric entailment is classical)

- $!\varphi \models !\psi$ *iff $\varphi \models_{classically} \psi$*

Fact 5 (Discourse entailment and equivalence)

1. $\varphi \, ; \, \psi \equiv \psi \, ; \, \varphi$ $\hspace{5cm}$ *(Order does not matter)*[9]

2. $\varphi \, ; \, \psi \models \varphi$ $\hspace{4cm}$ *(Sequencing behaves like conjunction)*

3. $!\varphi \, ; \, !\psi \equiv !(\varphi \wedge \psi)$ $\hspace{2.5cm}$ *(Assertoric sequencing is conjunction)*[10]

4. $?\varphi \models !\psi$ *iff* $\models !\psi$ $\hspace{3cm}$ *(Questions can only entail questions)*[11]

Fact 6 (Sentential entailment)

1. $!\neg\exists x.P(x) \models ?\exists x.P(x)$

2. $!P(a) \models ?\exists x.P(x)$

3. $!P(a) \not\models ?P(x)$

4. $!\forall x.(P(x) \leftrightarrow x = a) \models ?P(x)$

5. $?P(x) \models ?P(a)$

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[9]This means that there is nothing dynamic going on, as in Groenendijk (1999), though the semantics is formulated there as an update semantics, where typically order matters. That is why the subtitle of that paper is: *classical version*. A non-classical dynamic version, where order does matter, can be found in Groenendijk (1998).

[10] The third item indirectly points at the fact that, unlike for indicatives, the only way in which interrogatives can be conjoined in $\mathcal{L}_?$ is by sequencing them. The first and second item together imply that a sequence of interrogatives entails each interrogative in the sequence. So, sequencing has the features of conjunction. This is one reason why we introduced sequencing in the logical language: to have some means of mimicking conjunctions with interrogative conjuncts.

[11]If $\models !\psi$, then $!\psi$ is a tautology, and hence not informative, which means that by definition such $!\psi$ is a question. So, whether or not a discourse entails an indicative is fully independent of any interrogative occurring in that discourse. (See ten Cate and Shan (2007).)

3.6   The Inquisitive Hierarchy

Single worlds do not suffice to evaluate questions:[12]

Fact 7 (Intensionality of questions) *For all worlds* : $\{v\} \models ?\varphi$.

Proto-inquisitive semantics for questions is minimally intensional:

Proto-Fact 2 (Pair-distributivity) $s \models \varphi$ *iff for all* $v, w \in s$: $\{v, w\} \models \varphi$.

- Given pair-distributivity proto-support can just as well be defined relative to pairs of worlds: $v, w \models \varphi$.[13]

- In Ciardelli (2009); Ciardelli and Roelofsen (2011) it has been shown that such a pair-semantics is at the bottom of an inquisitive hierarchy, where only at the most general level, where support is defined relative to arbitrary states, we reach a real inquisitive semantics. This was how (general) inquisitive semantics came about.[14]

- Since proto-inquisitive semantics is essentially a pair-semantics, it does not count as a real inquisitive semantics.

3.7   Maximal supporting states: blocks in a partition

There is always at least one state that supports a sentence:

Fact 8 (The fatal state takes it all) $\emptyset \models \varphi$.

If a state supports a sentence then any more informed state does so as well:

Fact 9 (Persistence) *If* $s \models \varphi$ *and* $t \subseteq s$, *then* $t \models \varphi$.

- A maximal state that supports φ is a state s such that $s \models \varphi$, and for which there is no less informed state $t \supset s$ such that $t \models \varphi$.

Fact 10 (Maximality)

Every state that supports φ *is contained in a* maximal state *that supports it.*[15]

- Given persistence, the set of its maximal supporting states suffices to fully characterize the meaning of a sentence, the set of all states that support it.

Proto-Fact 3 (Maximal supporting states are blocks in a partition)

The maximal supporting states for φ *form a* partition *of* **info**(φ).[16]

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[12]There is a discussion in the literature concerning the issue whether questions are intensional or not. See Groenendijk and Stokhof (1997); Nelken (2002); Nelken and Shan (2006).

[13]In fact this was the format in which inquisitive semantics was originally formulated in Groenendijk (2009); Mascarenhas (2009); Sano (2009, 2011).

[14]Versions of general inquisitive semantics can already be found in Groenendijk (2008a). However, the author then erroneously believed that the general semantics can be reduced to a pair-semantics.

[15]This fact applies to both to the inquisitive propositional semantics presented in Section 1 and first order proto-inquisitive inquisitive semantics. For the current versions of first-order real inquisitive semantics it does not hold. See Ciardelli (2010, 2009).

[16]For any set A, $\{A\}$ also counts as a partition of A.

3.8   Propositions

Definition 13 (Propositions)

The proposition expressed by φ *is the set of maximal states that support it.*[17]

> *Notation:* $[\varphi]$ *is the proposition expressed by* φ.

> *We refer to the elements of* $[\varphi]$ *as* the possibilities for φ.

Proto-Fact 4 (A propositions is a partition of a set of worlds)

The proposition expressed by φ *is a partition of* $\mathsf{info}(\varphi)$, *and the possibilities for* φ *are the blocks in the partition.*

The proposition expressed by a sentence fully determines its meaning:

Fact 11 (Support and possibilities)

> $s \models \varphi$ *iff s is contained in a possibility for* φ

Example 7 $[!\exists x.P(x)] = \{\{v \in \omega \mid v(P) \neq \emptyset\}\}$. There is a single possibiity for $\exists x.Px$, the set of all worlds where the denotation of P is not empty.

Example 8 $[?\exists x.P(x)] = \{\{v \in \omega \mid v(P) \neq \emptyset\}, \{v \in \omega \mid v(P) = \emptyset\}\}$. There are two possibiities for $\exists x.P(x)$, the set of all worlds where the denotation of P is not empty, and the set of all worlds where the denotation of P is empty.

Example 9 $[?P(x)] = \{\{v \in \omega \mid v(P) = A\} \mid A \subseteq D\}$. There are as many possibilities for $?Px$ as there are possible denotations for P. For each subset of the domain, there is a possibility for $?P(x)$ that contains all worlds where that set of objects is the denotation of P.

Example 10 $[?R(x,y)] = \{\{v \in \omega \mid v(R) = A\} \mid A \subseteq D^2\}$. There are as many possibilities for $?P(x)$ as there are possible denotations for R. For each set of pairs of objects from the domain, there is a possibility for $?R(x,y)$ that contains all worlds where that set of pairs of objects is the denotation of R.

3.9   Propositions as proposals

- The notion of a proposition provides a uniform notion of meanings as sets of possibilities that applies to all discourses of our language, and hence to all single sentences, irrespective of their syntactic mood.

Sets of possibilities are just formal objects, sets of sets of possible worlds. We need an intuitive picture of what these formal objects are. How should we look upon propositions being such sets of sets of possible worlds?[18]

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[17]Like there are different versions of modal logic, S4, S5, etc., there is a range of variations concerning the notion of a proposition in inquisitive semantics. Possibilities as maximal supporting states is the most basic option. At the other side of the spectrum are arbitrary sets of possibilities, this notion of a proposition has been put to use in analyzing 'attentive *might*' (Ciardelli (2009); Ciardelli *et al.* (2009a)).

[18]The situation is not different for the classical notion of a proposition as a set of worlds. The accompanying intuitive picture there is that the worlds in the classical proposition expressed by a sentence φ are the worlds that are compatible with the information that φ provides.

- In uttering a sentence a speaker proposes one or more ways to update the common ground to the other participants in the conversation.

  - She invites the other participants to accept the information that the actual world is one that survives one of the proposed updates; and

  - She requests the other participants to provide the information that establishes one of the proposed updates.

- Each possibility α in the proposition expressed by a sentence φ correspond to a minimal way to provide information that establishes one of the updates that is proposed in uttering the sentence φ.

- Adding the information contained in a possibility α to the common ground cannot fail to lead to an update of the common ground that supports φ.

Fact 12 (Possibilities and support) *For all $\alpha \in [\varphi]$: $s \cap \alpha \models \varphi$.*

- Inquisitive semantics gives rise to a new notion of meaning, which is richer than that of a classical proposition.

- Propositions cover both informative and inquisitive content.

- The notion of a proposition in inquisitive semantics is inherently linked to the primary function of language to exchange information in a process of raising and resolving issues.

3.10   Some properties of propositions

Fact 13 (Inconsistency, informativeness, inquisitiveness) *For all $\varphi \in \mathcal{L}_?$:*

1. *φ is inconsistent iff $\mathsf{info}(\varphi) = \emptyset$ iff $[\varphi] = \{\emptyset\}$*

2. *φ is informative iff $\mathsf{info}(\varphi) \neq \omega$*

3. *φ is inquisitive iff $\mathsf{info}(\varphi) \notin [\varphi]$*

If $\mathsf{info}(\varphi) = \bigcup[\varphi] \notin [\varphi]$, then $[\varphi]$ should contain more than one possibility.

Fact 14 (Inquisitiveness)

  φ is inquisitive iff there are at least two possibilities for φ.

By definition, assertions are not inquisitive.

Fact 15 (Assertions)

1. *φ is an assertion iff there is a single possibility for φ.*

2. *$[\varphi] = \{\mathsf{info}(\varphi)\}$ iff φ is an assertion.*

3. *$[!\varphi] = \{|\varphi|\}$.*

Fact 16 (Tautologies and contradictions)

1. *φ is a tautology iff $[\varphi] = \{\omega\}$.*

2. *φ is a contradiction iff $[\varphi] = \{\emptyset\}$.*

Definition 14 (Polar sentences)

φ is a polar sentence iff there are exactly two possiblities for φ.

Fact 17 (Polar sentences are inquisitive)

If φ is a polar sentence, then φ is inquisitive.

Proto-Fact 5 (Polar sentences are questions)

if φ is a polar sentence, then φ is a question.

Fact 18 (Polar questions) *For all sentences $\varphi \in \mathcal{L}$:*

$$[?\varphi] = \{|\varphi|, |\neg\varphi|\} \textit{ iff } \varphi \textit{ is a contingent sentence in } \mathcal{L}.$$

Proto-Fact 6 (Some non-polar questions) *Let P be in $\mathcal{L}_P$:*

$$[?P(x_1, \ldots, x_n)] = \{\alpha \subseteq \omega_D \mid \textit{for all } w, v \in \alpha \colon w(P) = v(P)\}$$

There are as many possibilities for $?P(x_1, \ldots, x_n)$ as there are possible denotations for P.

4 Compliant responses

The core logical notion of inquisitive semantics is the notion of compliant responsehood, rather than the notion of entailment. Compliance is a very strict discourse relation. It is not intended as being *the only* relevant discourse relation. Its name is intended to reflect that: being compliant is not an absolute virtue, some situations call for non-compliant behavior. For a start, compliant responses will not cover *critical* responses like saying *No!* to a proposal that is inconsistent with your own information state. Such critical responses, signalling non-acceptance, are crucial in maintaining a common ground of a conversation.

4.1 Proto-inquisitive compliance

Unlike we did sofar, we start out with a definition of compliance that is restricted to proto-inquisitive semantics. Only at the end of this Section we will extend it to cover real inquisitive semantics. When we state a **Fact** rather than a **Proto-fact**, then in some cases the **Fact** only holds generally relative to the definition of compliance stated at the end of this Section.

Definition 15 (Proto-inquisitive compliance)

φ is a compliant response to ψ iff every possibility for φ is the union of a subset of the possibilities for ψ.

Example 11 (Compliance, partial answers and subquestions)

(16) $!P(a)$ is a compliant response to $?P(x)$

(17) $!\neg P(a)$ is a compliant response to $?P(x)$

(18) $?P(a)$ is a compliant response to $?P(x)$

(19) $!(P(a) \wedge P(b))$ is a compliant response to $?P(a)\,;\,?P(b)$

(20) $!(P(a) \wedge P(b))$ is not a compliant response to $?P(a)$

(21) $!(P(b) \rightarrow P(a))$ is a compliant response to $?P(a)\,;\,?P(b)$

(22) $!(P(b) \rightarrow P(a))$ is not a compliant response to $?P(a)$

Fact 19 (Properties of the compliance-relation)

1. φ is a compliant response to φ.

2. If φ is a compliant response to ψ, and ψ is a compliant response to χ, then φ is is a compliant response to χ.

3. If φ is a compliant response to ψ and vice versa, then $\varphi \equiv \psi$.

Fact 20 (Compliance)

1. φ is a compliant response to $!\psi$ iff $\varphi \equiv !\psi$.

2. If $?\varphi$ is a compliant response to ψ, then ψ is a question.

3. For polar sentences φ and ψ: φ is a compliant response to ψ iff $\varphi \equiv \psi$.

Remark 1 (Compliance and conditional questions)

The conditional question $p \rightarrow ?q$ and the questioned conditional $?(p \rightarrow q)$ are both polar sentences (polar questions). They are not logically equivalent. Hence, neither one counts as a compliant response to the other. And, none of the three proto-facts below applies in the case of conditional questions.

Proto-Fact 7 (Proto-inquisitive compliance)

1. $?\varphi$ is a compliant response to $?\psi$ iff $?\psi \models ?\varphi$.

2. $!\varphi$ is a compliant response to $?\psi$ iff $?\psi \models ?\varphi$.

3. $!\varphi$ is a compliant response to $?\psi$ iff $!\neg\varphi$ is a compliant response to $?\psi$.[19]

Remark 2 (Compliance and conditional questions, continued)

Contra 1: $?(p \rightarrow q) \models p \rightarrow ?q$, but according to the definition of compliance $p \rightarrow ?q$ is not a compliant response to $?(p \rightarrow q)$.

Contra 2: $(p \rightarrow q)$ is a compliant response to $p \rightarrow ?q$, but $p \rightarrow ?q \not\models ?(p \rightarrow q)$.

Contra 3: $(p \rightarrow q)$ is a compliant response to $p \rightarrow ?q$ but (whereas $p \rightarrow \neg q$ is) $\neg(p \rightarrow q)$, i.e., $p \wedge \neg q$ is not a compliant response to $p \rightarrow ?q$.

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[19] Unlike what this fact says, there are distinctive differences between positive and negative questions and answers. See Farkas and Bruce (2010); Farkas and Roelofsen (2011); Brasoveanu *et al.* (2011).

4.2   Comparative compliance

Definition 16 (Comparative informativeness and inquisitiveness)

1. *φ is* at least as informative as *ψ iff in every state where ψ is informative, φ is informative as well.*

2. *φ is* at least as inquisitive as *ψ iff in every state where ψ is inquisitive, φ is inquisitive as well.*

Fact 21 (Entailment, informativeness and inquisitiveness)

$\varphi \models \psi$ iff φ is at least as informative and at least as inquisitive as ψ.

Fact 22 (Compliance, informativeness and inquisitiveness)

If φ is a compliant response to ψ, then φ is at least as informative and at most as inquisitive as ψ.[20]

- Given this fact concerning compliance it comes natural to prefer among compliant responses those that are more informative and less inquisitive.

Definition 17 (Comparative compliance)

Let φ and χ be two non-equivalent compliant responses to ψ.

1. *φ is a* more compliant *response to ψ than χ iff φ is more informative or less inquisitive than χ.*

2. *φ is an* optimal compliant response *to ψ iff there is no more compliant response to ψ than φ.*

Fact 23 (Comparative compliance)

1. *Among non-equivalent non-inquisitive compliant responses (assertions) φ and χ to ψ comparative compliance prefers φ over χ iff $\varphi \models \chi$.*

2. *Among non-equivalent non-informative compliant responses (questions) φ and χ to ψ comparative compliance prefers φ over χ iff $\chi \models \varphi$.*

Example 12 (Optimal compliance and complete answers)

(23) $!\forall x.(P(x) \leftrightarrow (P(a) \vee P(b)))$ is an optimal compliant response to $?P(x)$

(24) $!\neg\exists x.P(x)$ is an optimal compliant response to $?P(x)$

(25) $!\forall x.P(x)$ is an optimal compliant response to $?P(x)$

Fact 24 (Optimal compliant responses are always assertions)

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[20]This fact holds without exception relative to proto-inquisitive semantics and proto-inquisitive compliance. We will meet exceptions for real inquisitive semantics in combination with proto-inquisitive compliance. But this fact will hold without exception again relative to the general notion of compliance to be defined at the end of this Section.

1. If ψ is an inquisitive sentence, then φ is an optimal compliant response to ψ iff φ is an assertion such that $\mathsf{info}(\varphi)$ is a possibility for ψ.

2. If ψ is a non-inquisitive sentence (assertion), then φ is an optimal compliant response to ψ iff φ is a compliant response to ψ, i.e., iff φ is an assertion such that $\varphi \equiv \psi$.

3. If ψ is a non-informative sentence (question), then among all non-informative compliant responses to ψ, the best ones are not inquisitive either, i.e., they are all equivalent with $\top$ (both a question and an assertion).

The last point deserves some attention. For informative responses it can easily happen that an optimal compliant response is not available to a responder, because Quality of a response requires that the state of the responder supports the informative content of her response. However, for non-informative responses, like a response with a counter-question, this informative Quality restriction can never apply. So, virtually, we can take this to mean that non-optimal inquisitive non-informative responses are never called for. You could just as well say that they are not compliant at all.[21]

4.3   Compliance and hybrid non-compliance

The use of the interjection *No* in (27), cannot be taken to signal denial of (26). It rather signals that (27) is a non-compliant response to (26).

| (26) | Alf or Bea will go to the party. | $P(a) \vee P(b)$ |
| (27) | No, Bea or Cor will go to the party. | $P(b) \vee P(c)$ |

Unlike (26) and (27), which are not related by entailment, neither classically nor inquisitively, (29) is entailed by (28), both classically and inquisitively.

| (28) | Alf or Bea will go to the party. | $P(a) \vee P(b)$ |
| (29) | No, Alf or Bea or Cor will go to the party. | $P(a) \vee P(b) \vee P(c)$ |

And in this case, too, the use of *No* in (29), cannot be taken to signal denial of (28). Again, it rather signals that (29) is a non-compliant response to (28), which it is according to the definition of proto-inquisitive compliance.

However, the use of *No* in (31) seems also in order, and seems to serve much the same function as in the previous examples.

| (30) | Alf or Bea or Cor will go to the party. | $P(a) \vee P(b) \vee P(c)$ |

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[21] In Groenendijk and Roelofsen (2009) we discarded tautological responses by a Quality requirement of significance. Then optimal counter-questions are polar subquestions of the initial question, and we provided an argumentation why this makes sense. However, as Dustin Tucker made clear in his comments on that paper at a workshop on questions in discourse and action, held in Michigan, October 2-4, 2009, the argumentation we presented applies with equal force to any arbitrary counter-question. From this perspective it seems preferable to have as a bottom line that the tautology is the most compliant non-informative response to any non-informative initiative.

(31)   No, Alf or Bea will go to the party. $P(a) \vee P(b)$

This time the definition of proto-inquisitive compliance does not predict that $P(a) \vee P(b)$ is not a compliant response to $P(a) \vee P(b) \vee P(c)$, on the contrary, it predicts that it is a compliant response.

Well, what the definition of proto-inquisitive compliance *does* predict is that $!(P(a) \vee P(b))$ is not a compliant response to $!(P(a) \vee P(b) \vee P(c))$. Of no two non-equivalent assertions one can be a compliant response to the other. But that also wrongly predicts that (33) is not a compliant response to (32).

(32)   Alf or Bea will go to the party. $P(a) \vee P(b)$

(33)   (*No) Alf will go. $P(a)$

The definition of proto-inquisitive compliance does make the right prediction here when, as in real inquisitive semantics, we interpret disjunction hybridly. But still it fails to do so in the previous case. This is remedied by adding an extra clause to the definition of compliance.[22]

Definition 18 (Compliance)

φ is a compliant response *to ψ iff*

1. *every possibility for φ is the union of a subset of the possibilities for ψ;*

2. *every possibility for ψ restricted to $\mathsf{info}(\varphi)$ is contained in a possibility for φ.*

Consider again $P(a) \vee P(b)$ in response to $P(a) \vee P(b) \vee P(c)$. The first condition is fulfilled, but the second condition is not: $|P(c)| \cap \mathsf{info}(P(a) \vee P(b) \vee P(c))$ is not a subset of $|P(a)|$ or $|P(b)|$, it overlaps with both.

The basic example by which the second clause in the definition of compliance is usually motivated is $?p$ in response to $?p \vee ?q$. The latter is a so-called choice-question. There are four possibilities for it: $|p|$, $|\neg p|$, $|q|$, $|\neg q|$. Each of the two possibilities for $?p$, $|p|$ and $|\neg p|$, is also a possibility for $?p \vee ?q$. So the first condition is fulfilled. Since $?p$ is not informative, the restriction to $\mathsf{info}(?\varphi)$ plays no role. And clearly, neither $|q|$ nor $|\neg q|$ is a subset of either $|p|$, $|\neg p|$. So, the second clause is not met: $?p$ is not a compliant response to $?p \vee ?q$.

Also, since $?p \models ?p \vee ?q$, and the two are not equivalent and not informative, $?p$ is more inquisitive than $?p \vee ?q$, which is not in accordance with the general feature that a compliant response φ to ψ is at most as inquisitive as ψ.

The same can be observed for $p \vee q$ and $p \vee q \vee r$. The one cannot be a compliant response to the other: $p \vee q$ is more inquisitive than $p \vee q \vee r$ and $p \vee q \vee r$ is less informative than $p \vee q$.

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[22]This is the definition of compliance as presented in Groenendijk and Roelofsen (2009). An algorithm to compute compliant responses for propositional inquisitive semantics can be found in Ciardelli *et al.* (2009b).

5 Conclusions

- Restating the semantics of Groenendijk (1999) using the semantic framework of inquisitive semantics went smoothly, and we think that the proto-inquisitive semantics we presented illuminates the system by providing a more standard support-semantics for it.

- From the perspective of real inquisitive semantics it is of some importance to have studied this system from which inquisitive semantics started, and to precisely delineate where the differences are. That also sheds light on the specific features of real inquisitive semantics as such.

- The most crucial difference is that while proto-inquisitive semantics is essentially a pair-semantics, due to its restrictive expressive power it escapes from the arguments in Ciardelli (2009); Ciardelli and Roelofsen (2011) against *inquisitive* pair-semantics, which is precisely why the proto-inquisitive semantics is not a *real* inquisitive semantics.

- We showed that in real, unlike in proto-inquisitive semantics conditional questions can easily be accommodated. This is achieved by the combination of a standard interpretation of implication and an inquisitive interpretation of disjunction. We used examples of conditional questions and hybrid disjunction to highlight the differences between proto- and real inquisitive semantics, in particular with respect to the core logical relation of compliance.

- We paid little or no attention to purely logical issues, but the logic of the first-order partition semantics of Groenendijk (1999) has been studied in detail in ten Cate and Shan (2007). The logic of propositional inquisitive semantics has also been fully explored in Ciardelli (2009); Ciardelli and Roelofsen (2011), and its algebraic properties have been studied in Roelofsen (2011) Given that we have found that the proto-inquisitive semantics lies just below the bottom of the inquisitive hierarchy, a combination of these logical studies may be a good starting point for logical investigations of first-order real inquisitive semantics.

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